Math 2280 - Practice Exam 2

University of Utah

Spring 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (25 points) *Population Models*

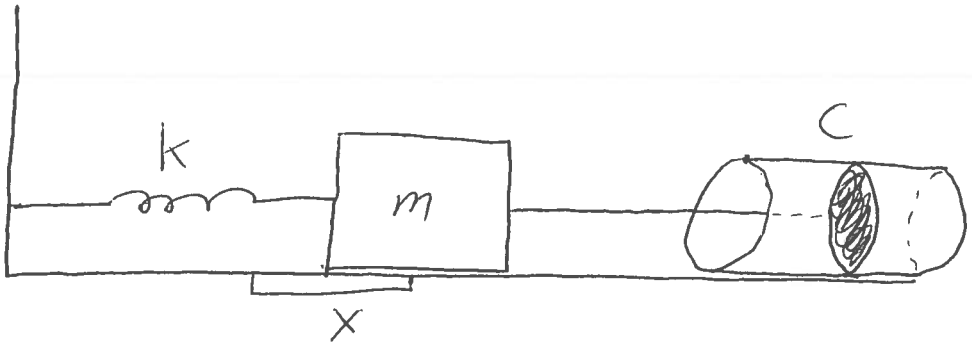
   For the population model

$$\frac{dP}{dt} = 3P(P-5)$$

   what are the equilibrium populations, and for each equilibrium population determine if it is a stable or unstable. Draw the corresponding phase diagram. Then, use separation of variables to solve the population model exactly with the initial population $P(0) = 2$.

More room for problem 1, if you need it.

2. (20 points) *Mechanical Systems*



For the mass-spring-dashpot system drawn[1] above, find the equation that describes its motion with the parameters:

$$m = 3;$$

$$c = 30;$$

$$k = 63;$$

and initial conditions:

$$x_0 = 2; \qquad v_0 = 2.$$

Is the system overdamed, underdamped, or critically damped?

[1] Poorly.

More room for problem 2, if you need it.

3. (20 points) *Higher Order Linear Homogeneous Differential Equations with Constant Coefficients*

   Find the general solution to the differential equation:

$$y^{(4)} + 2y^{(3)} + y'' - 12y' + 8 = 0.$$

   Hint: The polynomial $x^4 + 2x^3 + x^2 - 12x + 8$ has $x = 1$ as a root of multiplicity 2.

4. (15 points) *Wronskians*

   Use the Wronskian to prove that the functions

$$f(x) = e^x \qquad g(x) = e^{2x} \qquad h(x) = e^{3x}$$

   are linearly independent on the real line $\mathbb{R}$.

5. (20 points) *Nonhomogeneous Linear Equations*
   Find the solution to the linear ODE:

$$y'' - y' - 2y = 3x + 4,$$

   with initial conditions

$$y(0) = \frac{7}{4} \qquad\qquad y'(0) = \frac{3}{2}.$$

More room for problem 5, if you need it.