Math 2280 - Exam 2

University of Utah

Spring 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (25 points) Population Models

For the population model

$$\frac{dP}{dt} = 3P(5-P)$$

what are the equilibrium populations, and for each equilibrium popuation determine if it is a stable or unstable. Draw the corresponding phase diagram. Then, use separation of variables to solve the population model exactly with the initial population P(0) = 8. More room for problem 1, if you need it.

2. (20 points) Euler's Method

Use Euler's method with a step size h = 1 to calculate an approximate value for y(3), where y(x) is the solution to the initial value problem

$$\frac{dy}{dx} = -3x^2y,$$
$$y(0) = 3.$$

More room for problem 2, if you need it.

3. (20 points) *Higher Order Linear Homogeneous Differential Equations with Constant Coefficients*

Find the general solution to the differential equation:

$$y^{(3)} + y'' - y' - y = 0.$$

4. (15 points) Wronskians

Use the Wronskian to prove that the functions

$$f(x) = e^x$$
 $g(x) = \cos x$ $h(x) = \sin x$

are linearly independent on the real line $\mathbb R.$

5. (20 points) *Nonhomogeneous Linear Equations* Find the solution to the linear ODE:

$$y'' + 3y' + 2y = e^{-x},$$

with initial conditions

$$y(0) = 0$$
 $y'(0) = 3.$

More room for problem 5, if you need it.