## Math 2280 - Practice Exam 1

University of Utah

Spring 2013

Solution Name: \_

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

- 1. (15 Points) Differential Equation Basics
  - (a) (5 points) What is the order of the differential equation given below?

$$(y'')^2 + 2xy' - \sin(x)y^3 = 14x^2 + e^x$$

(b) (5 points) Is the differential equation given below linear?

$$y^{(3)} + 2x^{2}y'' - 4\sin(x)y' + e^{x}y = x^{4} + x^{2}e^{x}$$

(c) (5 points) On what intervals are we guaranteed a unique solution exists for the differential equation below?

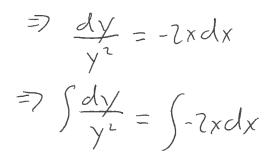
$$y' + \ln(x)y = \frac{x+3}{x^2-1}$$
  
 $\ln(x)$  is defined for  $x > 0$ , and  $\frac{x+3}{x^2-1}$  is  
defined for  $x \neq \pm 1$ . So, we're guaranteed a  
unique solution exists for  $An$  initial value  
 $(a,b)$  if  $a \in (0,1)$  or  $a > 1$ .

## 2. (25 points) Separable Equations

Find the general solution to the differential equation given below.

 $\frac{dy}{dx} + 2xy^2 = 0$ 

$$\frac{dy}{dx} = -2xy^2$$



$$= 7 - \frac{1}{y} = -x^{2} + C$$
$$= 7 - \frac{1}{y} = \frac{1}{x^{2} + C}$$

## 3. (25 points) Substitution Methods

Find the general solution to the differential equation given below.

$$y' = y + y^{3}$$

$$y' - y = y^{3}$$
Bernoulli equation.  

$$v = y^{-2} \qquad \frac{dv}{dx} = -2y^{-3} \frac{dy}{dx}$$

$$= y' = -\frac{y^{3}}{2}v'$$

$$y' - y = y^{3} = \frac{y'}{y^{3}} - \frac{1}{y^{2}} = 1$$

$$= \frac{y'}{y^{3}} = -\frac{v'}{2} - \frac{1}{y^{2}} = -v$$

$$= \frac{v'}{2} - v = 1 = \frac{v'}{2} + \frac{1}{2}v = -2$$

$$e^{2x}v' + \frac{1}{2}e^{2x}v = -2e^{2x}$$

$$= \frac{d}{dx}(e^{2x}v) = -2e^{2x} = \frac{e^{2x}v}{2} - \frac{1}{2}e^{-\frac{1}{2}x} = \frac{1}{2}e^{-\frac{1}{2}e^{-\frac{1}{2}x}} = \frac{1}{2}e^{-\frac{1}{2$$

## 4. (10 points) Existence and Uniqueness

For what initial conditions y(a) = b does the following differential equation have a unique solution in an interval around *a*?

$$\frac{dy}{dx} = \sqrt{x-y}$$
  
 $f(x_{1Y}) = \sqrt{x-y}$  is defined for  $x \ge \gamma$ .  
 $\frac{\partial f}{\partial \gamma} = -\frac{1}{2\sqrt{x-\gamma}}$  is defined for  $x \ge \gamma$ .  
Also, both continuous for  $x \ge \gamma$ .  
So, for  $\frac{1}{\sqrt{x-\gamma}}$  if  $a > b$  a unique  
so lution exists in an interval around a.

5. (25 points) First-Order Linear Equations

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Find a solution to the initial value problem given below, and explain, if it's true, why the solution is unique.

$$xy' - y = x \qquad \qquad y(1) = 7.$$

$$y' - \frac{y}{x} = 1 \qquad e^{\int -\frac{dy}{x}} = e^{-\ln x + i} = \frac{c}{x}$$

$$\frac{d}{dx} \left( \frac{y}{x} \right) - \frac{y}{x^{2}} = \frac{1}{x}$$

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$$\frac{y}{x} = \ln (x) + c$$

$$\frac{y(x)}{x} = x \ln(x) + (1) = 7 \qquad y' - \left(\frac{1}{x}\right)y = 1$$

$$\frac{y(x)}{x} = x \ln(x) + 7x \qquad Bo + h - \frac{1}{x} and 1 are$$

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$$\frac{y(x)}{x} = \frac{1}{x} and x = 1$$