

Math 2280 - Practice Exam 1

University of Utah

Spring 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (15 Points) *Differential Equation Basics*

- (a) (5 points) What is the order of the differential equation given below?

$$(y'')^2 + 2xy' - \sin(x)y^3 = 14x^2 + e^x$$

- (b) (5 points) Is the differential equation given below linear?

$$y^{(3)} + 2x^2y'' - 4\sin(x)y' + e^xy = x^4 + x^2e^x$$

- (c) (5 points) On what intervals are we guaranteed a unique solution exists for the differential equation below?

$$y' + \ln(x)y = \frac{x+3}{x^2-1}$$

2. (25 points) *Separable Equations*

Find the general solution to the differential equation given below.

$$\frac{dy}{dx} + 2xy^2 = 0$$

3. (25 points) *Substitution Methods*

Find the general solution to the differential equation given below.

$$y' = y + y^3$$

4. (10 points) *Existence and Uniqueness*

For what initial conditions $y(a) = b$ does the following differential equation have a unique solution in an interval around a ?

$$\frac{dy}{dx} = \sqrt{x - y}$$

5. (25 points) *First-Order Linear Equations*

Find a solution to the initial value problem given below, and explain, if it's true, why the solution is unique.

$$xy' - y = x \qquad y(1) = 7.$$