Math 2280 - Exam 1

University of Utah

Spring 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (15 Points) *Differential Equation Basics*

   (a) (5 points) What is the order of the differential equation given below?[1]

$$y'x^3 + (y'')^2 + y^{(3)}y^2 \sin(x) = 14x^5 + 7x^2 - e^{-x^2}$$

   (b) (5 points) Is the differential equation given below linear?

$$xy'y + 2xy^2 = \cos e^x$$

   (c) (5 points) On what intervals are we guaranteed a unique solution exists for the differential equation below?

$$y' + \frac{y}{x} = \frac{x+3}{x^2-1}$$

[1] Extra credit - Solve this differential equation! Just kidding. Do not attempt to solve it.

2. (25 points) *Separable Equations*

   Find the general solution to the differential equation given below.

$$\frac{dy}{dx} - 3\sqrt{xy} = 0$$

3. (30 points) *Exact Equations*

   Find the solution to the initial value problem given below.[2]

$$\frac{dy}{dx} = -\frac{\cos x + \ln y}{\frac{x}{y} + e^y}$$

   with initial condition $y(0) = 5$.

[2]The title of this problem is a hint.

4. (30 points) *First-Order Linear Equations*

   Find a solution to the initial value problem given below, and give the interval upon which you know the solution is unique.

$$(1+x)y' + y = \cos x \qquad\qquad y(0) = 1.$$