Math 2280 - Assignment 4

Dylan Zwick

Spring 2013

Section 2.4 - 1, 5, 9, 26, 30

Section 3.1 - 1, 16, 18, 24, 39

Section 3.2 - 1, 10, 16, 24, 31

Section 2.4 - Numerical Approximation: Euler's Method

2.4.1 Apply Euler's method twice to approximate the solution to the initial value problem below on the interval $[0, \frac{1}{2}]$, first with step size $h = .25$, then with step size $h = 0.1$. Compare the three-decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value $y(\frac{1}{2})$ of the actual solution, also given below.

$$y' = -y,$$

$$y(0) = 2;$$

$$y(x) = 2e^{-x}.$$

2.4.5 Apply Euler's method twice to approximate the solution to the initial value problem below on the interval $[0, \frac{1}{2}]$, first with step size $h = .25$, then with step size $h = 0.1$. Compare the three-decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value $y(\frac{1}{2})$ of the actual solution, also given below.

$$y' = y - x - 1,$$

$$y(0) = 1;$$

$$y(x) = 2 + x - e^x.$$

2.4.9 Apply Euler's method twice to approximate the solution to the initial value problem below on the interval $[0, \frac{1}{2}]$, first with step size $h = .25$, then with step size $h = 0.1$. Compare the three-decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value $y(\frac{1}{2})$ of the actual solution, also given below.

$$y' = \frac{1}{4}(1 + y^2),$$

$$y(0) = 1;$$

$$y(x) = \tan\left(\frac{1}{4}(x + \pi)\right).$$

2.4.26 Suppose the deer population $P(t)$ in a small forest initially numbers 25 and satisfies the logistic equation

$$\frac{dP}{dt} = 0.0225P - 0.0003P^2$$

(with t in months.) Use Euler's method with a programmable calculator or computer to approximate the solution for 10 years, first with step size $h = 1$ and then with $h = .5$, rounding off approximate P-values to integrals numbers of deer. What percentage of the limiting population of 75 deer has been attained after 5 years? After 10 years?

More room for problem 2.4.26.

2.4.30 Apply Euler's method with successively smaller step sizes on the interval $[0, 2]$ to verify empirically that the solution of the initial value problem

$$\frac{dy}{dx} = y^2 + x^2, \quad y(0) = 0$$

has vertical asymptote near $x = 2.003147$.

More room for problem 2.4.30.

Section 3.1 - Second-Order Linear Equations

3.1.1 Verify that the functions y_1 and y_2 given below are solutions to the second-order ODE also given below. Then, find a particular solution of the form $y = c_1y_1 + c_2y_2$ that satisfies the given initial conditions. Primes denote derivatives with respect to x.

$$y'' - y = 0;$$

$$y_1 = e^x \quad y_2 = e^{-x};$$

$$y(0) = 0 \quad y'(0) = 5.$$

3.1.16 Verify that the functions y_1 and y_2 given below are solutions to the second-order ODE also given below. Then, find a particular solution of the form $y = c_1 y_1 + c_2 y_2$ that satisfies the given initial conditions. Primes denote derivatives with respect to x.

$$x^2 y'' + xy' + y = 0;$$

$$y_1 = \cos(\ln x), \quad y_2 = \sin(\ln x);$$

$$y(1) = 2, \quad y'(1) = 3.$$

3.1.18 Show that $y = x^3$ is a solution of $yy'' = 6x^4$, but that if $c^2 \neq 1$, then $y = cx^3$ is not a solution.

3.1.24 Determine if the functions

$$f(x) = \sin^2 x, \quad g(x) = 1 - \cos(2x)$$

are linearly dependent on the real line $\mathbb{R}$.

3.1.30 **(a)** Show that $y_1 = x^3$ and $y_2 = |x^3|$ are linearly independent solutions on the real line of the equation $x^2y'' - 3xy' + 3y = 0$.

(b) Verify that $W(y_1, y_2)$ is identically zero. Why do these facts not contradict Theorem 3 from the textbook?

Section 3.2 - General Solutions of Linear Equations

3.2.1 Show directly that the given functions are linearly dependent on the real line. That is, find a non-trivial linear combination of the given functions that vanishes identically.

$$f(x) = 2x, \quad g(x) = 3x^2, \quad h(x) = 5x - 8x^2.$$

3.2.10 Use the Wronskian to prove that the given functions are linearly independent.

$$f(x) = e^x, \quad g(x) = x^{-2}, \quad h(x) = x^{-2} \ln x; \, x > 0.$$

3.2.16 Find a particular solution to the third-order homogeneous linear equation given below, using the three linearly independent solutions given below.

$$y^{(3)} - 5y'' + 8y' - 4y = 0;$$

$$y(0) = 1, \quad y'(0) = 4, \quad y''(0) = 0;$$

$$y_1 = e^x, \quad y_2 = e^{2x}, \quad y_3 = xe^{2x}.$$

3.2.24 Find a solution satisfying the given initial conditions for the differential equation below. A complementary solution y_c, and a particular solution y_p are given.

$$y'' - 2y' + 2y = 2x;$$

$$y(0) = 4 \quad y'(0) = 8;$$

$$y_c = c_1 e^x \cos x + c_2 e^x \sin x \quad y_p = x + 1.$$

3.2.31 This problem indicates why we can impose *only* n initial conditions on a solution of an nth-order linear differential equation.

(a) Given the equation

$$y'' + py' + qy = 0,$$

explain why the value of $y''(a)$ is determined by the values of $y(a)$ and $y'(a)$.

(b) Prove that the equation

$$y'' - 2y' - 5y = 0$$

has a solution satisfying the conditions

$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = C,$$

if and only if $C = 5$.

More room for problem 3.2.31.