Math 2280 - Assignment 14

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Section 9.4 - 1, 2, 3, 19, 20

Section 9.5 - 1, 3, 5, 7, 9

Section 9.6 - 1, 3, 5, 7, 14

Section 9.4 - Applications of Fourier Series

9.4.1 - Find the steady periodic solution, $x_{sp}(t)$, of the differential equation below.

$$x'' + 5x = F(t),$$

where $F(t)$ is the function of period 2π such that $F(t) = 3$ if $0 < t < \pi$, and $F(t) = -3$ if $\pi < t < 2\pi$.

9.4.2 - Find the steady periodic solution, $x_{sp}(t)$, of the differential equation below.

$$x'' + 10x = F(t),$$

where $F(t)$ is the even function of period 4 such that $F(t) = 3$ if $0 < t < 1$, and $F(t) = -3$ if $1 < t < 2$.

9.4.3 - Find the steady periodic solution, $x_{sp}(t)$, of the differential equation below.

$$x'' + 3x = F(t),$$

where $F(t)$ is the odd function of period 2π such that $F(t) = 2t$ if $0 < t < \pi$.

9.4.19 - Suppose the functions $f(t)$ and $g(t)$ are periodic with periods P and Q respectively. If the ratio P/Q of their periods is a rational number, show that the sum $f(t) + g(t)$ is a periodic function.

9.4.20 - If p/q is irrational, prove that the function $f(t) = \cos pt + \cos qt$ is not a periodic function. *Suggestions*: Show that the assumption $f(t + L) = f(t)$ would (upon substituting $t = 0$) imply that p/q is rational.

Section 9.5 - Heat Conduction and Separation of Variables

9.5.1 - Solve the boundary value problem $u_t = 3u_{xx}$, $0 < x < \pi$, $t > 0$; $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 4\sin 2x$.

9.5.3 - Solve the boundary value problem $u_t = 2u_{xx}$, $0 < x < 1$, $t > 0$; $u(0,t) = u(1,t) = 0$, $u(x,0) = 5\sin \pi x - \frac{1}{5}\sin 3\pi x$.

9.5.5 - Solve the boundary value problem $u_t = 2u_{xx}$, $0 < x < 3$, $t > 0$; $u_x(0,t) = u_x(3,t) = 0$, $u(x,0) = 4\cos\frac{2}{3}\pi x - 2\cos\frac{4}{3}\pi x$.

9.5.7 - Solve the boundary value problem $3u_t = u_{xx}$, $0 < x < 2$, $t > 0$; $u_x(0,t) = u_x(2,t) = 0$, $u(x,0) = \cos^2 2\pi x$.

9.5.9 - Solve the boundary value problem $10u_t = u_{xx}$, $0 < x < 5$, $t > 0$; $u(0,t) = u(5,t) = 0$, $u(x,0) = 25$.

Section 9.6 - Vibrating Strings and the One-Dimensional Wave Equation

9.6.1 - Solve the boundary value problem $y_{tt} = 4y_{xx}$, $0 < x < \pi$, $t > 0$; $y(0,t) = y(\pi,t) = 0$, $y(x,0) = \frac{1}{10}\sin 2x$, $y_t(x,0) = 0$.

9.6.3 - Solve the boundary value problem $4y_{tt} = y_{xx}$, $0 < x < \pi$, $t > 0$; $y(0,t) = y(\pi,t) = 0$, $y(x,0) = y_t(x,0) = \frac{1}{10}\sin x$.

9.6.5 - Solve the boundary value problem $y_{tt} = 25y_{xx}$, $0 < x < 3$, $t > 0$; $y(0,t) = y(3,t) = 0$, $y(x,0) = \frac{1}{4}\sin \pi x, y_t(x,0) = 10\sin 2\pi x.$

9.6.7 - Solve the boundary value problem $y_{tt} = 100y_{xx}$, $0 < x < 1$, $t > 0$; $y(0,t) = y(1,t) = 0$, $y(x,0) = 0, y_t(x,0) = x$.

9.6.14 - Given the differentiable odd period $2L$ function $F(x)$, show that the function

$$y(x,t) = \frac{1}{2}[F(x+at) + F(x-at)]$$

satisfies the conditions $y(0,t) = y(L,t) = 0$, $y(x,0) = F(x)$, and $y_t(x,0) = 0$.