Math 2280 - Assignment 12

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Section 8.3 - 1, 8, 15, 18, 24

Section 8.4 - 1, 6, 8, 9, 14

Section 8.3 - Regular Singular Points

8.3.1 - Determine whether $x = 0$ is an ordinary point, a regular singular point, or an irregular singular point for the differential equation

$$xy'' + (x - x^3)y' + (\sin x)y = 0.$$

If it is a regular singular point, find the exponents of the differential equation (the solutions to the indicial equation) at $x = 0$.

8.3.8 - Determine whether $x = 0$ is an ordinary point, a regular singular point, or an irregular singular point for the differential equation

$$(6x^2 + 2x^3)y'' + 21xy' + 9(x^2 - 1)y = 0.$$

If it is a regular singular point, find the exponents of the differential equation (the solutions to the indicial equation) at $x = 0$.

8.3.15 - If $x = a \neq 0$ is a singular point of a second-order linear differential equation, then the substitution $t = x - a$ transforms it into a differential equation having $t = 0$ as a singular point. We then attribute to the original equation at $x = a$ the behavior of the new equation at $t = 0$. Classify (as regular or irregular) the singular points of the differential equation

$$(x-2)^2 y'' - (x^2-4)y' + (x+2)y = 0.$$

8.3.18 - Find two linearly independent Frobenius series solutions (for $x > 0$) to the differential equation

$$2xy'' + 3y' - y = 0.$$

More room for Problem 8.3.18, if you need it.

8.3.24 - Find two linearly independent Frobenius series solutions (for $x > 0$) to the differential equation

$$3x^2y'' + 2xy' + x^2y = 0.$$

More room for Problem 8.3.24, if you need it.

Section 8.4 - Method of Frobenius: The Exceptional Cases

8.4.1 - Either apply the method from Example 1 in the textbook to find two linearly independent Frobenius series solutions, or find one such solution and show (as in Example 2 from the textbook) that a second such solution does not exist for the differential equation:

$$xy'' + (3 - x)y' - y = 0.$$

More room for Problem 8.4.1, if you need it.

8.4.6 - Either apply the method from Example 1 in the textbook to find two linearly independent Frobenius series solutions, or find one such solution and show (as in Example 2 from the textbook) that a second such solution does not exist for the differential equation:

$$2xy'' - (6 + 2x)y' + y = 0.$$

More room for Problem 8.4.6, if you need it.

8.4.8 - Either apply the method from Example 1 in the textbook to find two linearly independent Frobenius series solutions, or find one such solution and show (as in Example 2 from the textbook) that a second such solution does not exist for the differential equation:

$$x(1-x)y'' - 3y' + 2y = 0.$$

More room for Problem 8.4.8, if you need it.

8.4.9 - For the differential equation

$$xy'' + y' - xy = 0,$$

first find the first four nonzero terms in a Frobenius series solution. Then use the reduction of order technique to find the logarithmic term and the first three nonzero terms in a second linearly independent solution.

More room for Problem 8.4.9, if you need it.

8.4.14 - For the differential equation

$$x^2y'' + x(1+x)y' - 4y = 0,$$

first find the first four nonzero terms in a Frobenius series solution. Then use the reduction of order technique to find the logarithmic term and the first three nonzero terms in a second linearly independent solution.

More room for Problem 8.4.14, if you need it.