Math 2280 - Assignment 10

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Section 7.4 - 1, 5, 10, 19, 31

Section 7.5 - 1, 6, 15, 21, 26

Section 7.6 - 1, 6, 11, 14, 15

Section 7.4 - Derivatives, Integrals, and Products of Transforms

7.4.1 - Find the convolution $f(t) * g(t)$ of the functions

$$f(t) = t, \quad g(t) = 1.$$

7.4.5 - Find the convolution $f(t) * g(t)$ of the functions

$$f(t) = g(t) = e^{at}.$$

7.4.10 - Apply the convolution theorem to find the inverse Laplace transform of the function

$$F(s) = \frac{1}{s^2(s^2 + k^2)}.$$

7.4.19 - Find the Laplace transform of the function

$$f(t) = \frac{\sin t}{t}.$$

7.4.31 - Transform the given differential equation to find a nontrivial solution such that $x(0) = 0$.

$$tx'' - (4t + 1)x' + 2(2t + 1)x = 0.$$

Section 7.5 - Periodic and Piecewise Continuous Input Functions

7.5.1 - Find the inverse Laplace transform $f(t)$ of the function

$$F(s) = \frac{e^{-3s}}{s^2}.$$

7.5.6 - Find the inverse Laplace transform $f(t)$ of the function

$$F(s) = \frac{se^{-s}}{s^2 + \pi^2}.$$

7.5.15 - Find the Laplace transform of the function

$$f(t) = \sin t \text{ if } 0 \leq t \leq 3\pi;\ f(t) = 0 \text{ if } t > 3\pi.$$

7.5.21 - Find the Laplace transform of the function

$$f(t) = t \text{ if } t \leq 1;\ f(t) = 2 - t \text{ if } 1 \leq t \leq 2;\ f(t) = 0 \text{ if } t > 2.$$

7.5.26 - Apply Theorem 2 to show that the Laplace transform of the sawtooth function $f(t)$ pictured below is

$$F(s) = \frac{1}{as^2} - \frac{e^{-as}}{s(1-e^{-as})}.$$

More room for Problem 7.5.26, if you need it.

Inpulses and Delta Functions

7.6.1 - Solve the initial value problem

$$x'' + 4x = \delta(t);$$

$$x(0) = x'(0) = 0,$$

and graph the solution $x(t)$.

7.6.6 - Solve the initial value problem

$$x'' + 9x = \delta(t - 3\pi) + \cos 3t;$$

$$x(0) = x'(0) = 0,$$

and graph the solution $x(t)$.

7.6.11 - Apply Duhamel's principle to write an integral formula for the solution of the initial value problem

$$x'' + 6x' + 8x = f(t);$$

$$x(0) = x'(0) = 0.$$

7.6.14 - Verify that $u'(t-a) = \delta(t-a)$ by solving the problem

$$x' = \delta(t-a);$$

$$x(0) = 0$$

to obtain $x(t) = u(t-a)$.

7.6.15 - This problem deals with a mass m on a spring (with constant k) that receives an impulse $p_0 = mv_0$ at time $t = 0$. Show that the initial value problems

$$mx'' + kx = 0;$$

$$x(0) = 0,\ x'(0) = v_0$$

and

$$mx'' + kx = p_0\delta(t);$$

$$x(0) = 0,\ x'(0) = 0$$

have the same solution. Thus the effect of $p_0\delta(0)$ is, indeed, to impart to the particle an initial momentum p_0.

More space, if you need it, for Problem 7.6.15.