Math 2280 - Assignment 1

Dylan Zwick

Spring 2013

Section 1.1 - 1, 12, 15, 20, 45

Section 1.2 - 1, 6, 11, 15, 27, 35, 43

Section 1.3 - 1, 6, 9, 11, 15, 21, 29

Section 1.1 - Differential Equations and Mathematical Models

1.1.1 Verify by substitution that the given function is a solution of the given differential equation. Throughout these problems, primes denote derivatives with respect to x.

$$y' = 3x^2; \qquad y = x^3 + 7$$

1.1.12 Verify by substitution that the given function is a solution of the given differential equation.

$$x^2y'' - xy' + 2y = 0; \qquad y_1 = x\cos(\ln x), \quad y_2 = x\sin(\ln x).$$

1.1.15 Substitute $y = e^{rx}$ into the given differential equation to determine all values of the constant r for which $y = e^{rx}$ is a solution of the equation

$$y'' + y' - 2y = 0$$

1.1.20 First verify that $y(x)$ satisfies the given differential equation. Then determine a value of the constant C so that $y(x)$ satisfies the given initial condition.

$$y' = x - y; \qquad y(x) = Ce^{-x} + x - 1, \qquad y(0) = 10$$

1.1.45 Suppose a population P of rodents satisfies the differential equation $dP/dt = kP^2$. Initially, there are $P(0) = 2$ rodents, and their number is increasing at the rate of $dP/dt = 1$ rodent per month when there are $P = 10$ rodents. How long will it take for this population to grow to a hundred rodents? To a thousand? What's happening here?

Section 1.2 - Integrals as General and Particular Solutions

1.2.6 Find a function $y = f(x)$ satisfying the given differential equation and the prescribed initial condition.

$$\frac{dy}{dx} = 2x + 1; \qquad y(0) = 3.$$

1.2.6 Find a function $y = f(x)$ satisfying the given differential equation and the prescribed initial condition.

$$\frac{dy}{dx} = x\sqrt{x^2 + 9} \qquad\qquad y(-4) = 0.$$

1.2.11 Find the position function $x(t)$ of a moving particle with the given acceleration $a(t)$, initial position $x_0 = x(0)$, and initial velocity $v_0 = v(0)$.

$$a(t) = 50,$$

$$v_0 = 10,$$

$$x_0 = 20.$$

1.2.15 Find the position function $x(t)$ of a moving particle with the given acceleration $a(t)$, initial position $x_0 = x(0)$, and initial velocity $v_0 = v(0)$.

$$a(t) = 4(t+3)^2,$$

$$v_0 = -1,$$

$$x_0 = 1.$$

1.2.27 A ball is thrown straight downward from the top of a tall building. The initial speed of the ball is $10m/s$. It strikes the ground with a speed of $60m/s$. How tall is the building?

1.2.35 A stone is dropped from rest at an initial height h above the surface of the earth. Show that the speed with which it strikes the ground is $v = \sqrt{2gh}$.

1.2.43 Arthur Clark's *The Wind from the Sun* (1963) describes Diana, a spacecraft propelled by the solar wind. Its aluminized sail provides it with a constant acceleration of $0.001g = 0.0098 m/s^2$. Suppose this spacecraft starts from rest at time $t = 0$ and simultaneously fires a projectile (straight ahead in the same direction) that travels at one-tenth of the speed $c = 3 \times 10^8 m/s$ of light. How long will it take the spacecraft to catch up with the projectile, and how far will it have traveled by then?

1.3.1 and 1.3.6 See Below

In Problems 1 through 10, we have provided the slope field of the indicated differential equation, together with one or more solution curves. Sketch likely solution curves through the additional points marked in each slope field.

1. $\dfrac{dy}{dx} = -y - \sin x$

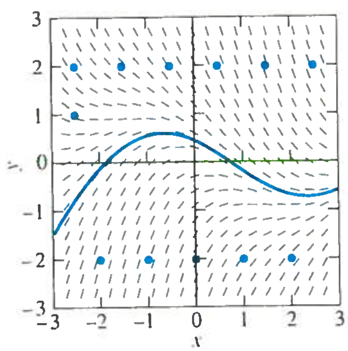

FIGURE 1.3.15.

2. $\dfrac{dy}{dx} = x + y$

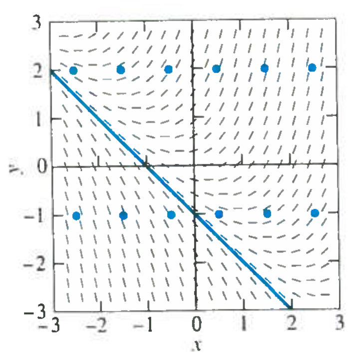

FIGURE 1.3.16.

3. $\dfrac{dy}{dx} = y - \sin x$

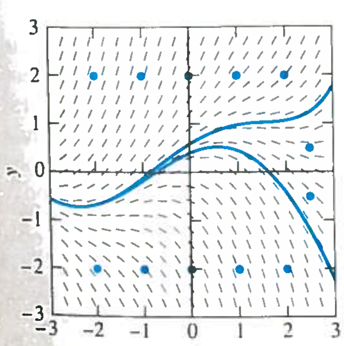

FIGURE 1.3.17.

4. $\dfrac{dy}{dx} = x - y$

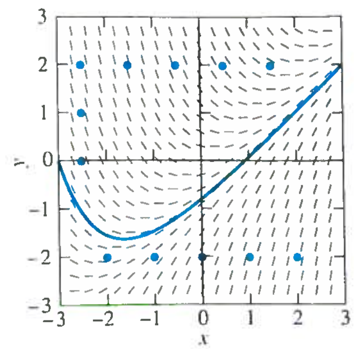

FIGURE 1.3.18.

5. $\dfrac{dy}{dx} = y - x + 1$

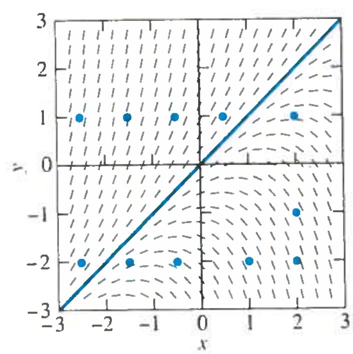

FIGURE 1.3.19.

6. $\dfrac{dy}{dx} = x - y + 1$

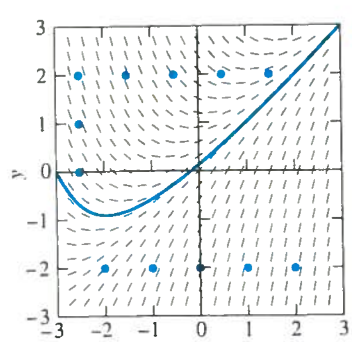

FIGURE 1.3.20.

1.3.9 See Below

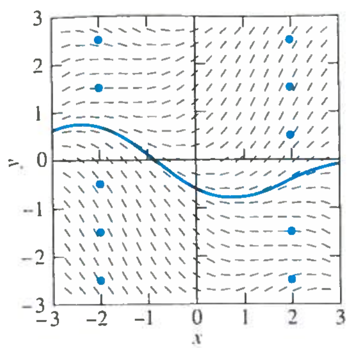

FIGURE 1.3.21.

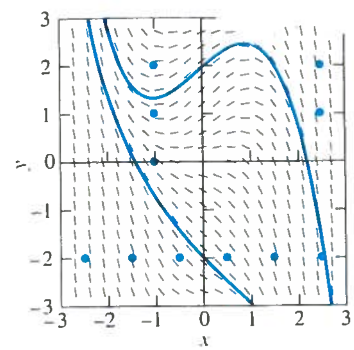

FIGURE 1.3.24.

8. $\dfrac{dy}{dx} = x^2 - y$

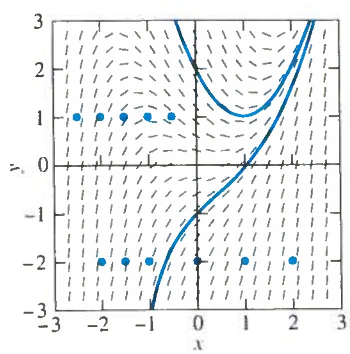

FIGURE 1.3.22.

9. $\dfrac{dy}{dx} = x^2 - y - 2$

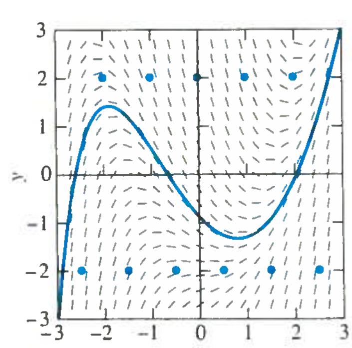

FIGURE 1.3.23.

In Problems 11 through 20, determine whether Theorem 1 does or does not guarantee existence of a solution of the given initial value problem. If existence is guaranteed, determine whether Theorem 1 does or does not guarantee uniqueness of that solution.

11. $\dfrac{dy}{dx} = 2x^2y^2; \quad y(1) = -1$

12. $\dfrac{dy}{dx} = x \ln y; \quad y(1) = 1$

13. $\dfrac{dy}{dx} = \sqrt[3]{y}; \quad y(0) = 1$

14. $\dfrac{dy}{dx} = \sqrt[3]{y}; \quad y(0) = 0$

15. $\dfrac{dy}{dx} = \sqrt{x - y}; \quad y(2) = 2$

16. $\dfrac{dy}{dx} = \sqrt{x - y}; \quad y(2) = 1$

17. $y\dfrac{dy}{dx} = x - 1; \quad y(0) = 1$

18. $y\dfrac{dy}{dx} = x - 1; \quad y(1) = 0$

19. $\dfrac{dy}{dx} = \ln(1 + y^2); \quad y(0) = 0$

20. $\dfrac{dy}{dx} = x^2 - y^2; \quad y(0) = 1$

In Problems 21 and 22, first use the method of Example 2 to construct a slope field for the given differential equation. Then sketch the solution curve corresponding to the given initial condition. Finally, use this solution curve to estimate the desired value of the solution $y(x)$.

21. $y' = x + y, \quad y(0) = 0; \quad y(-4) = ?$

22. $y' = y - x, \quad y(4) = 0; \quad y(-4) = ?$

1.3.11 Determine whether Theorem 1 does or does not guarantee existence of a solution of the given initial value problem. If existence is guaranteed, determine whether Theorem 1 does or does not guarantee uniqueness of that solution.

$$\frac{dy}{dx} = 2x^2y^2 \qquad\qquad y(1) = -1.$$

1.3.15 Determine whether Theorem 1 does or does not guarantee existence of a solution of the given initial value problem. If existence is guaranteed, determine whether Theorem 1 does or does not guarantee uniqueness of that solution.

$$\frac{dy}{dx} = \sqrt{x - y} \qquad y(2) = 2.$$

1.3.21 First use the method of Example 2 from the textbook to construct a slope field for the given differential equation. Then sketch the solution curve corresponding to the given initial condition. Finally, use this solution curve to estimate the desired value of the solution $y(x)$.

$$y' = x + y, \qquad y(0) = 0; \qquad y(-4) = ?$$

1.3.29 Verify that if c is a constant, then the function defined piecewise by

$$y(x) = \begin{cases} 0 & x \le c, \\ (x-c)^3 & x > c \end{cases}$$

satisfies the differential equation $y' = 3y^{\frac{2}{3}}$ for all x. Can you also use the "left half" of the cubic $y = (x-c)^3$ in piecing together a solution curve of the differential equation? Sketch a variety of such solution curves. Is there a point (a, b) of the xy-plane such that the initial value problem $y' = 3y^{\frac{2}{3}}, y(a) = b$ has either no solution or a unique solution that is defined for all x? Reconcile your answer with Theorem 1.

More room for Problem 1.3.29