Refresher Course Math 1050 and 1060 Practice Problems Set 2 Fall 2007

1.) Find the domain of each of the following:

a.) $3x^4 - 8x + 12$

All real numbers

b.) $\sqrt{x+6}$

$x \geq -6$

c.) $\frac{x+5}{x-9}$

$x \neq 9$

2.) Write in simplest form:

a.) $\frac{w^2}{2w^3}$

$\frac{1}{2w}$

b.) $\frac{x^2 - y^2}{x^2 + 2xy + y^2}$

$\frac{(x-y)(x+y)}{(x+y)^2} = \frac{x-y}{x+y}$

c.) $\frac{5m - 25}{m^2 - 25}$

$\frac{5(m-5)}{(m-5)(m+5)} = \frac{5}{m+5}$

d.) $\frac{x+3}{-x-3}$

-1

e.) $\frac{-x+3}{-3+x}$

-1

3.) Perform the indicated operation:

a.) $\dfrac{2x^2+x-6}{x^2+4x-5}\cdot\dfrac{x^3-3x^2+2x}{4x^2-6x}$

$$\frac{(2x-3)(x+2)}{(x+5)(x-1)}\cdot\frac{x(x-2)(x-1)}{2x(2x-3)}=\boxed{\frac{(x+2)(x-2)}{2(x+5)}}$$

b.) $\dfrac{4x^2+12x}{2x+1}\div\dfrac{-x^3-3x^2}{4x^2-1}$

$$=\frac{4x(x+3)}{(2x+1)}\cdot\frac{(2x+1)(2x-1)}{-x^2(x+3)}=\boxed{-\frac{4(2x-1)}{x}}$$

c.) $\dfrac{2k}{k^2-5k+4}+\dfrac{3}{k^2-1}$

$$\frac{2k}{(k-4)(k-1)}+\frac{3}{(k-1)(k+1)}=\frac{2k(k+1)+3(k-4)}{(k-4)(k-1)(k+1)}$$

d.) $\dfrac{4y}{y^2-1}-\dfrac{5}{y^2+2y+1}$

$$=\frac{2k^2+5k-12}{(k-4)(k-1)(k+1)}=\boxed{\frac{(2k-3)(k+4)}{(k-4)(k-1)(k+1)}}$$

$$\frac{4y}{(y+1)(y-1)}-\frac{5}{(y+1)^2}=\frac{4y(y+1)-5(y-1)}{(y+1)^2(y-1)}=\frac{4y^2-y+5}{(y+1)^2(y-1)}$$

e.) $\dfrac{-2}{p+1}+\dfrac{4p}{p^2-1}$

$$=\boxed{\frac{4y^2-y+5}{(y+1)^2(y-1)}}$$

$$\frac{-2}{p+1}+\frac{4p}{p^2-1}=\frac{-2(p-1)}{(p+1)(p-1)}+\frac{4p}{(p+1)(p-1)}$$

$$=\frac{-2p+2+4p}{(p+1)(p-1)}=\frac{2p+2}{(p+1)(p-1)}=\boxed{\frac{2}{p-1}}$$

4.) Simplify the following complex fraction: $\dfrac{\frac{x}{4}-\frac{3}{2}}{2-\frac{3}{x}}$

$$\frac{\left(\frac{x-6}{4}\right)x}{\left(2-\frac{3}{x}\right)x} = \frac{\frac{x^2-6x}{4}}{2x-3} = \boxed{\frac{x(x-6)}{4(2x-3)}}$$

5.) Simplify $\dfrac{2x^2(x-1)^{1/2}-5(x-1)^{-1/2}}{x-1}$

$$= \frac{\left(2x^2\sqrt{x-1}-\frac{5}{\sqrt{x-1}}\right)\sqrt{x-1}}{x-1\ (\sqrt{x-1})}$$

$$= \frac{2x^2(x-1)-5}{(x-1)^{3/2}} = \boxed{\frac{2x^3-2x^2-5}{(x-1)^{3/2}}}$$

6.) $2x^3 = 2x^2+12x$

Divide both sides by $2x$, noting $x=0$ is a solution

$x^2 = x+6 \quad x^2-x-6=0 \Rightarrow (x-3)(x+2)=0$

7.) $\dfrac{3x^2+6x}{\sqrt{x+2}} = \sqrt{x+2}$

So, solutions are $\boxed{x=0,3,-2}$

Can also write $\boxed{2x(x-3)(x+2)=0}$

$3x^2+6x = x+2$

$3x^2+5x-2=0$

$\boxed{(3x-1)(x+2)=0}$

So, $\boxed{x=\frac{1}{3},-2}$

8.) $\frac{2x^2+5x-3}{2x-1}=x+3$

$\frac{(2x-1)(x+3)}{(2x-1)} = x+3$

$(2x-1)(x+3) = (x+3)(2x-1)$

So, solutions are all real numbers except $x=\frac{1}{2}$, as that's not in the domain.

9.) $\sqrt{3x+4}=x-2$

$3x+4 = (x-2)^2$

$3x+4 = x^2-4x+4$

$0 = x^2-7x = x(x-7)$

So, $x=0, 7$ are possible solutions.

Check

$x=7$

$\sqrt{3(7)+4} = 7-2$

$\sqrt{25} = 5$

$5=5$ ✓

$x=0$

$\sqrt{4} = 0-2$

$2=-2$ ✗

So, $x=7$ is only solution

10.) $\frac{2x(x+1)-\frac{(x+1)^2}{x}}{\sqrt{x-1}}=0$

Domain is $x>1$

$2x(x+1) - \frac{(x+1)^2}{x} = 0$

$2x^2(x+1) - (x+1)^2 = 0$

$(x+1)(2x^2-(x+1)) = 0$

$(x+1)(2x+1)(x-1) = 0$

So, $x=-1, -\frac{1}{2}, 1$

None are in the domain!

So, ~~solut~~ no real solutions.

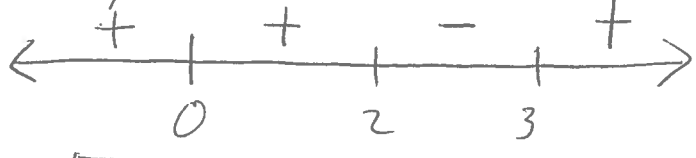
11.) $x^4 < 5x^3-6x^2$

$x^4-5x^3+6x^2 < 0$

$x^2(x^2-5x+6)$

$x^2(x-3)(x-2) < 0$

Sign line



So, $2 < x < 3$ $(2,3)$

12.) $\dfrac{2x(x+1)-\dfrac{(x+1)^2}{x}}{\sqrt{x-1}} \geq 0$

$\sqrt{x-1}$ must always be positive.

$$\frac{2x^2(x+1)-(x+1)^2}{x} = \frac{(x+1)(2x+1)(x-1)}{x} \geq 0$$

Critical points at $x = -1, 0, -\frac{1}{2}, 1$ none of which are in the domain. So, it's positive on all its domain $\boxed{x > 1}$ $(1, \infty)$

13.) $|15x+8| \leq 3$

$-3 \leq 15x+8 \leq 3$

$15x+8 \leq 3 \quad x \leq -\frac{1}{3}$

$15x \leq -5$

$-3 \leq 15x+8$

$-11 \leq 15x$

$\frac{-11}{15} \leq x$

So, $\boxed{-\frac{11}{15} \leq x \leq -\frac{1}{3}}$

$\left[-\frac{11}{15}, -\frac{1}{3}\right]$

14). $|15x+8| > 3$

Complement of 13)

$\boxed{x < -\frac{11}{15} \text{ or } x > -\frac{1}{3}}$

$\left(-\infty, -\frac{11}{15}\right) \cup \left(-\frac{1}{3}, \infty\right)$

15.) $3(x-5)(x+1)+10 = 2(x-1)(x+2)+x^2-14x-1$

$3(x^2-4x-5)+10 = 2(x^2+x-2)+x^2-14x-1$

$3x^2-12x-15+10 = 2x^2+2x-4+x^2-14x-1$

$3x^2-12x-5 = 3x^2-12x-5$

$\Rightarrow \boxed{0=0}$ So, true for $\boxed{\text{all real numbers.}}$