# Math 2280 - Exam 3

University of Utah

Spring 2009

**Name:** _____

**Defective Eigenvalues** - Solve the system of ODEs:

$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 18 & 7 & 4 \\ -27 & -9 & -5 \end{pmatrix} \mathbf{x}.$$

(10 points).

Continued...

**Matrix Exponentials** - Calculate $e^{At}$ for the matrix:

$$A = \begin{pmatrix} 3 & 0 & -3 \\ 5 & 0 & 7 \\ 3 & 0 & -3 \end{pmatrix}.$$

(5 points).

**Undetermined Coefficients** - Apply the method of undetermined coefficients to find a particular solution for the system of ODEs:

$$x' = x - 5y + 2\sin t,$$

$$y' = x - y - 3\cos t.$$

(5 points).

Continued...

**Laplace Transforms** - Calculate the Laplace transform of the function:

$$f(t) = t^2$$

directly from the definition of the Laplace transform. (5 points).

**Solving ODEs with Laplace Transforms** - Use Laplace transform methods to solve the initial value problem:

$$x'' - 6x' + 8x = 2;$$

$$x(0) = x'(0) = 0.$$

(10 points)

Continued...

**Convolutions and Products** - Using the definition of convolution calculate the convolution product:

$$f(t) * g(t)$$

where $f(t) = t^2$ and $g(t) = t$. (7 points)

Calculate the Laplace transform $\mathcal{L}(f(t) * g(t))$. (3 points)

**Delta Functions** - Solve the initial value problem:

$$x'' + 2x' + x = \delta(t) - \delta(t-2);$$

$$x(0) = x'(0) = 2.$$

(10 points).

Continued...

You may find the following formulas useful:

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt$$

$$f(t) * g(t) = \int_0^t f(\tau) g(t-\tau) d\tau$$

$$\mathcal{L}(f(t) * g(t)) = \mathcal{L}(f(t)) \cdot \mathcal{L}(g(t))$$

$$\mathcal{L}(u(t-a) f(t-a)) = e^{-as} F(s)$$

$$\mathcal{L}(t^n e^{at}) = \frac{n!}{(s-a)^{n+1}}$$