Math 2280 - Exam 2

University of Utah

Spring 2009

Name: _____

Existance and Uniqueness - State whether we're certain (based on our existance and uniqueness theorem for linear differential equations) a unique solution exists for the following differential equations on the given interval. Explain why. (5 points)

1. (1 point)

$$y'' - x(y')^2 + e^x y = 2x^2 - 5;$$

for all $x \in \mathbb{R}$.

2. (2 points)

$$xy'' - e^x y' + \cos xy = 25x^3;$$

for all $x > 1$.

3. (2 points)

$$xy'' - e^x y' + \cos xy = 25x^3;$$

for all $x < 1$.

Linear Differential Equations with Constant Coefficients (10 points)

1. Find the general solution to the following homogeneous differential equation: (3 points)

$$y'' - y' - 6y = 0$$

2. Use this result to calculate the general solution to the nonhomogeneous differential equation: (4 points)

$$y'' - y' - 6y = 2x + e^{-2x}$$

3. Find the unique solution to the following initial value problem: (3 points)

$$y'' - y' - 6y = 2x + e^{-2x}$$

$$y(0) = 2, y'(0) = \frac{7}{15}$$

Wronskians - Calculate the Wronskian for the following sets of functions, and determine if the functions are linearly independent. If the functions are not linearly independent, demonstrate a non-trivial linear combination that equals 0. (5 points)

1. (2 points)

$$y_1 = e^{3x}$$

$$y_2 = xe^{3x}$$

2. (3 points)

$$y_1 = \sin 2x$$

$$y_2 = \sin x \cos x$$

Converting to First-Order Systems - Convert the following system of equations into an equivlent system of first-order equations: (5 points)

$$x^{(3)} = x'' - 2x' + 5y' + 2x + 1$$

$$y'' = x' + 5x - 14y'$$

Circuits Calculate the steady periodic current for the circuit pictured below: (10 points)

with the following parameters: $R = 200\Omega$, $L = 5H$, $C = .001F$, and $E(t) = 100\sin(10t)V$.

Continued...

First-Order Systems Solve the system of first-order differential equations given below: (10 points)

$$x_1' = 3x_1 + x_2 + x_3$$

$$x_2' = -5x_1 - 3x_2 - x_3$$

$$x_3' = 5x_1 + 5x_2 + 3x_3$$

Method of Undetermined Coefficients Find the form of the particular solution (but don't calculate the constants) for the nonhomogeneous linear differential equation given below using the method of undetermined coefficients: (5 points)

$$y^{(4)} - 2y'' + 3y' - 10y = x^3 e^{-x} \cos 4x$$