Math 2280 - Exam 1

University of Utah

Spring 2009

Name: _____

1. **Basics** (5 points)

   For the differential equation:

   $$e^{2x-4}(y'')y - \sin xy' = 7$$

   What is the order of this differential equation? (2 point)

   Is this differential equation linear? (1 point)

   For the differential equation:

   $$x^2 y' - xy = e^x$$

   What is the order of the differential equation? (1 point)

   Is the differential equation linear? (1 point)

2. **Separable Differential Equations**

   Find solutions for the given initial value problems:

   (a) (5 points)

   $$\frac{dy}{dx} = 6e^{2x-y}$$

   $$y(0) = 0$$

(b) (5 points)

$$\frac{dy}{dx} = 3x^2(y^2 + 1)$$

$$y(0) = 1$$

Note - $\int \frac{dx}{1 + x^2} = \arctan x + C$

3. **First-Order Linear Differential Equations** (10 points)

Solve the given first-order linear differential equations.

(a) (5 points) Find the general solution.

$$y' - 2xy = e^{x^2}$$

(b) (5 points) Find the unique solution for the given initial condition.

$$y' = 1 + x + y + xy$$

$$y(0) = 0$$

4. **Existence and Uniqueness** (5 points)

Does the differential equation:

$$\frac{dy}{dx} = y^2 e^{xy} + \sin x - x^3 + 2$$

Have a unique solution for any initial condition $y(a) = b$, where a and b are arbitrary real numbers? Justify your answer.

5. **Population Models** (5 points)

The time rate of change of a rabbit population P is proportional to the square root of P. At time $t = 0$ (months) the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be one year later?

6. **Autonomous Differential Equations** (5 points)

   For the autonomous differential equation:

   $$\frac{dx}{dt} = x^2 - 5x + 4$$

   Find all critical points, draw the corresponding phase diagram, and indicate whether the critical points are stable, unstable, or semi-stable.

7. **Euler's Method**

   Use Euler's method to fill in the rest of the table for the differential equation below with the given initial conditions and step size.

   Note - The y_n in the table below represent your estimated values given by Euler's method.

$$y' = 3x^2y^2;$$

$$y(0) = 1;$$

$$h = .5.$$

n	x	y_n
0	0	1
1	.5	
2		
3		