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Math 2280

Assignment #4 Solutions

2.4.1 Apply Euler's method twice to approximate the solution on the interval [0, ½], first with step size .25 then with step size .1. Compare the 3 decimal place solutions of the approximations with the actual value at $x = \frac{1}{2}$.

$y' = -y \qquad y(0) = 2 \qquad y(x) = 2e^{-x}$

For $h = .25$

$y_0 = 2 \qquad x = 0$

$y_1 = y_0 + (.25)(-2) \qquad x = .25$

$= 2 - \frac{1}{2} = \frac{3}{2}$

$y_2 = \frac{3}{2} + (.25)(-\frac{3}{2}) \qquad x = .5$

$= \frac{3}{2} - \frac{3}{8} = \boxed{\frac{9}{8}}$

For $h = .1$

$y_0 = 2$

$y_1 = 2 + (.1)(-2) \qquad x = .1$

$= 2 - .2 = 1.8$

$y_2 = 1.8 + (.1)(-1.8) \qquad x = .2$

$= 1.8 - .18 = 1.62$

$y_3 = 1.62 + (.1)(-1.62) \qquad x = .3$

$= 1.458$

$y_4 = 1.458 + (.1)(-1.458) \qquad x = .4$

$= 1.3122$

$y_5 = 1.3122 + (.1)(-1.3122) \qquad x = .5$

$= 1.18098$

$$\begin{array}{r} 31220 \\ -13122 \\ \hline 18098 \end{array}$$

Now, the exact value of y at $x = .5$ is:

$y(.5) = 2e^{-.5} = 1.213$

So, for $h = .25$ off by:

$9/8 = 1.125$ difference: $1.213 - 1.125 = \boxed{.088}$

While for $h = .1$ off by:

$1.213 - 1.181 = \boxed{.032}$

2.4.5. $y' = y - x - 1$ $y(0) = 1$ $y(x) = 2 + x - e^x$

<u>$h = .25$</u>

$y_0 = 1$ $x = 0$

$y_1 = 1 + (.25)(1 - 0 - 1)$ $x = .25$

$= 1$

$y_2 = 1 + (.25)(1 - .25 - 1)$

$= 1 - \frac{1}{16} = \boxed{\frac{15}{16}}$

<u>$h = .1$</u>

$y_0 = 1$ $x = 0$

$y_1 = 1 + (.10)(1 - 0 - 1)$ $x = .1$

$= 1$

$y_2 = 1 + (.1)(1 - .1 - 1)$ $x = .2$

$= \cancel{1.01}\ .99$

$y_3 = \cancel{1.01 + (.1)(1.01 - .2 - 1)}\ \cancel{x = .3}$

$.99 + (.1)(.99 - .2 - 1)$

$= \cancel{.991} = .969$

$y_4 = \cancel{.991 + (.1)(.991 - .3 - 1)}$ $x = .4$

$.969 + (.1)(.969 - .3 - 1)$

$= \cancel{.9601} = .1359$

$y_5 = \cancel{.9601 + .1(.9601 - .4 - 1)}$ $\quad x = .9$

$.9359 + .1(.9359 - .4 - 1)$

$= \cancel{.91611}$ $.88949$

Actual value at $x = .9$

$y(.9) = 2 + .9 - e^{.9} = .891$

So, $h = .25$ off by:

$.938 - .851 = .087$

$h = .1$ off by:

$.889 - .851 = .038$

~~.916 - .891 = .065~~

Not a huge diference, but $h = .1$ is the better approximation

4.9. $y' = \frac{1}{4}(1+y^2)$, $y(0) = 1$ $\quad y(x) = \tan(\frac{1}{4}(x+\pi))$

$y_0 = 1$ $\quad x = 0$

$y_1 = 1 + \frac{1}{4}(\frac{1}{4}(1+1^2))$ $\quad x = .25$

$= 1 + \frac{1}{8} = \frac{9}{8}$

$y_2 = \frac{9}{8} + \frac{1}{4}(\frac{1}{4}(1+(\frac{9}{8})^2))$ $\quad \cancel{x = .25}$ $x = .5$

$= 1.27$

$h = .1$

$y_0 = \cancel{0}\,1 \qquad x = 0$

$y_1 = 1 + (.1)\left(\frac{1}{4}(1+1^2)\right) \qquad x = .1$

$= 1 + .05 = 1.05$

$y_2 = 1.05 + (.1)\left(\frac{1}{4}(1+(1.05)^2)\right) \qquad x = .2$

$= \cancel{1.26}\ 1.10$

$y_3 = 1.10 + (.1)\left(\frac{1}{4}(1+(1.10)^2)\right) \qquad x = .3$

$= 1.158$

$y_4 = 1.158 + (.1)\left(\frac{1}{4}(1+1.158^2)\right) \qquad x = .4$

$= 1.217$

$y_5 = \cancel{1.279}\ 1.217 + (.1)\left(\frac{1}{4}(1+1.217^2)\right) \qquad x = .5$

$= 1.279$

Now, the actual value is:

$y(.5) = \tan\left(\frac{1}{4}(.5+\pi)\right) = 1.287$

So,

$h = .25$ off by:

$1.287 - 1.27 = .017.$

$h = .1$ off by:

$1.287 - 1.279 = .008.$

4.26. Suppose the deer population $P(t)$ in a small forest initially numbers 25 and satisfies the logistic equation

$$\frac{dP}{dt} = 0.0225P - 0.0003P^2$$

(with t in months). Use Euler's method with a programmable calculator or computer to approximate the solution for 10 years, first with step size $h=1$ and then with $h=.5$, rounding off approximate P-values to integral number of deer. What percentage of the limiting population of 75 deer has been obtained after 5 years? After 10 years?

Note: If we round every step, the population doesn't change.

$h = 1$ month			$h = .5$ month	
t	P	Rounded P	P	Rounded P
0	25	25	25	25
1 month	25.375	25	25.376	25
2 months	25.753	26	25.754	26
1 year	29.667	30	29.675	30
5 years	49.389	49	49.390	49
10 years	66.180	66	66.235	66

Percentage after 5 years: 65%
Percentage after 10 years: 88%

Note: $h=1$ and $h=.5$ almost identical!

2.4.30 Apply Euler's method with successively smaller step sizes on the interval [0, 2] to verify emperically that the solution of the initial value problem

$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 0$$

has a vertical asymptote near $x = 2.003147$.

x-value	y-values $h=.5$	$h=.1$	$h=.01$	$h=.001$
.5	0	.030	.041	.042
1	.125	.401	.344	.350
1.5	.633	1.213	1.479	1.518
2	1.958	5.892	28.393	142.627

We see as h gets small as we approach x=2 we get very large y values. This is caused by the asymptote.

Note: I had to write a Java program to do this.

3.1.1. Verify y_1 and y_2 are solutions, and then find constants $c_{1,2}$ such that $y = c_1 y_1 + c_2 y_2$ satisfies the given initial conditions

$$y'' - y = 0 \qquad y_1 = e^x \quad y_2 = e^{-x} \quad \begin{array}{l} y(0)=0 \\ y'(0)=5 \end{array}$$

$$y_1' = e^x$$
$$y_1'' = e^x$$

$$y_1'' - y_1 = e^x - e^x = 0$$

$$y_2' = -e^{-x}$$
$$y_2'' = e^{-x}$$

$$y_2'' - y_2 = e^{-x} - e^{-x} = 0$$

So, both y_1 and y_2 work.

$$y = c_1 y_1 + c_2 y_2 = c_1 e^x + c_2 e^{-x}$$
$$y' = c_1 e^x - c_2 e^{-x}$$

So,
$$y(0) = c_1 + c_2 = 0$$
$$y'(0) = c_1 - c_2 = 5$$
$$\Rightarrow c_1 = \tfrac{5}{2} \quad c_2 = -\tfrac{5}{2}$$

So,
$$\boxed{y(x) = \tfrac{1}{2}(5e^x - 5e^{-x})}$$

3.1.16. $x^2y'' + xy' + y = 0$ $\qquad y_1 = \cos(\ln x)$

$\qquad\qquad\qquad\qquad\qquad\qquad y_2 = \sin(\ln x)$

$y(1) = 2 \qquad y'(1) = 3$

$$y_1' = \frac{-\sin(\ln x)}{x} \qquad y_1'' = \frac{-\cos(\ln x) + \sin(\ln x)}{x^2}$$

$$x^2y_1'' + xy_1' + y_1 = -\cos(\ln x) + \sin(\ln x) - \sin(\ln x) + \cos(\ln x) = 0$$

So, checks out for y_1.

$$y_2' = \frac{\cos(\ln x)}{x} \qquad y_2'' = \frac{-\sin(\ln x) - \cos(\ln x)}{x^2}$$

$$x^2y_2'' + xy_2' + y_2 = -\sin(\ln x) - \cos(\ln x) + \cos(\ln x) + \sin(\ln x) = 0$$

So, checks out for y_2.

$$y = c_1y_1 + c_2y_2 = c_1\cos(\ln x) + c_2\sin(\ln x)$$

$$y(1) = c_1\cos(\ln(1)) + c_2\sin(\ln(1)) = c_1 = 2$$

$$y'(x) = \frac{-2\sin(\ln x)}{x} + \frac{c_2\cos(\ln x)}{x}$$

$$y'(1) = \frac{-2\sin(\ln 1)}{1} + \frac{c_2\cos(\ln 1)}{1} = c_2 = 3$$

$$\Rightarrow \boxed{y(x) = 2\cos(\ln(x)) + 3\sin(\ln(x))}$$

3.1.18 Show that $y = x^3$ is a solution of $yy'' = 6x^4$, but that if $c^2 \neq 1$, then $y = cx^3$ is not a solution.

Suppose $y = cx^3$ then $y' = 3cx^2$ and $y'' = 6cx$

So,

$$yy'' = 6c^2x^4 = 6x^4$$

if and only if $c^2 = 1$, and so $c = \pm 1$.

So,

$$y = x^3 \quad \text{or} \quad y = -x^3$$

work, but no other c works.

3.1.24. Determine if $f(x)$ and $g(x)$ are linearly independent on the real line $\mathbb{R}$.

$f(x) = \sin^2 x \qquad g(x) = 1 - \cos 2x$

Now, $\sin^2 x = \dfrac{1-\cos 2x}{2}$ (basic trig. identity)

So, $2g(x) = f(x)$

and they are not linearly independent

Other proof:

$$W(f,g) = \begin{vmatrix} \sin^2 x & 1-\cos 2x \\ 2\sin x \cos x & 2\sin 2x \end{vmatrix} = 2\sin^2 x \sin 2x - 2\sin x\cos x(1-\cos 2x) = 0 \text{ when } x = 0.$$

3.1.39 Find a general solution to the ODE:

$$4y'' + 4y' + y = 0$$

Characteristic Equation:

$$4r^2 + 4r + 1 = 0 \Rightarrow (2r+1)^2$$

So, $r = -\frac{1}{2}$ is a root of order 2.

So, the general solution will be:

$$\boxed{y(x) = c_1 e^{-x/2} + c_2 x e^{-x/2}}$$

.2.1 Show that the following functions are linearly dependent on $\mathbb{R}$.

$f(x) = 2x, \quad g(x) = 3x^2, \quad h(x) = 5x - 8x^2$

$$6h(x) + 16g(x) - 15f(x)$$

$$= 6(5x - 8x^2) + 16(3x^2) - 15(2x)$$
$$= 30x - 48x^2 + 48x^2 - 30x$$
$$= \boxed{0}$$

3.2.10 Use the Wronskian to prove the following functions are linearly independent

$$f(x) = e^x \qquad g(x) = x^{-2} \qquad h(x) = x^{-2}\ln x \qquad x > 0$$

$$W(f,g,h) = \begin{vmatrix} e^x & x^{-2} & x^{-2}\ln x \\ e^x & -2x^{-3} & x^{-3} - 2x^{-3}\ln x \\ e^x & 6x^{-4} & -3x^{-4} - 2x^{-4} + 6x^{-4}\ln x \end{vmatrix}$$

$$\cancel{= e^x \begin{vmatrix} -\frac{2}{x^2} & \frac{1}{x^3} - \frac{2\ln x}{x^3} \\ \frac{6}{x^4} & -5 + 6\ln x \end{vmatrix}}$$

Evaluate at $x = 1$ to get

$$W(f,g,h)(1) = \begin{vmatrix} e & 1 & 0 \\ e & -2 & 1 \\ e & 6 & -5 \end{vmatrix}$$

$$= e(10-6) - e(-5-0) + e(1-0)$$

$$= 4e + 5e + e = 10e \neq 0$$

So, linearly independent

3.2-16 Find a particular solution to the given ODE with given linearly independent homogenous solutions that satisfy the given initial conditions

$$y^{(3)} - 5y'' + 8y' - 4y = 0; \quad y(0) = 1 \quad y'(0) = 4 \quad y''(0) = 0;$$

$$y_1 = e^x \quad y_2 = e^{2x} \quad y_3 = xe^{2x}$$

$$y(x) = c_1 y_1 + c_2 y_2 + c_3 y_3$$

$$= c_1 e^x + c_2 e^{2x} + c_3 x e^{2x}$$

$$y(0) = c_1 + c_2 \cancel{+ c_3}$$

$$y'(x) = c_1 e^x + 2c_2 e^{2x} + 2c_3 x e^{2x} + c_3 e^{2x}$$

$$y'(0) = c_1 + 2c_2 + c_3$$

$$y''(x) = c_1 e^x + 4c_2 e^{2x} + 4c_3 x e^{2x} + 4c_3 e^{2x}$$

$$y''(0) = c_1 + 4c_2 + 4c_3$$

$$\Rightarrow \begin{matrix} c_1 + c_2 = 1 \\ c_1 + 2c_2 + c_3 = 4 \\ c_1 + 4c_2 + 4c_3 = 0 \end{matrix} \quad \Rightarrow \quad \begin{matrix} c_2 + c_3 = 3 \\ 3c_2 + 4c_3 = -1 \end{matrix} \qquad c_2 = 3 - c_3$$

$$3(3 - c_3) + 4c_3 = -1 \Rightarrow 9 + c_3 = -1 \Rightarrow c_3 = -10$$

$$\Rightarrow c_2 = 13 \qquad c_1 = -12$$

So,

$$\boxed{y(x) = -12e^x + 13e^{2x} - 10xe^{2x}}$$

3.2.24 Find a solution to the given ODE that satisfies the initial conditions with the given y_c and y_p.

$y''-2y'+2y=2x \quad y(0)=4 \quad y'(0)=8$

$y_c = c_1 e^x \cos x + c_2 e^x \sin x \qquad y_p = x+1$

$y = c_1 e^x \cos x + c_2 e^x \sin x + x + 1$

$y' = -c_1 e^x \sin x + c_1 e^x \cos x + c_2 e^x \cos x + c_2 e^x \sin x + 1$
$= (-c_1 + c_2) e^x \sin x + (c_1 + c_2) e^x \cos x + 1$

$y(0) = c_1 + 1 = 4 \quad \Rightarrow c_1 = 3$
$y'(0) = c_1 + c_2 + 1 = 8 \Rightarrow 3 + c_2 + 1 = 8 \Rightarrow c_2 = 4$

So,

$\boxed{y(x) = 3e^x \cos x + 4e^x \sin x + x + 1}$

3.2.31 This problem indicates why we can only impose n initial conditions on the solution of an n-th order linear differential equation.

a) Given the equation

$$y''+py'+qy=0$$

explain why the value of $y''(a)$ is determined by the values of $y(a)$ and $y'(a)$.

Well,

$$y''(a)=-p(a)y'(a)-q(a)y(a).$$

So, if $y(a)$, $p(a)$, $y'(a)$, and $q(a)$ are known then $y''(a)$ is determined.

b) Prove that the equation

$$y''-2y'-5y=0$$

has a solution satisfying the conditions

$$y(0)=1 \quad y'(0)=0 \quad \text{and} \quad y''(0)=C$$

if and only if $C=5$.

Well,

$$y''(0)=2y'(0)+5y(0)=2(0)+5(1)=5$$

So, we must have $y''(0)=5$.

2.5.1 Apply the improved Euler method to approximate this solution on the interval [0, 0.5] with step size $h = .1$. Construct a table showing four-decimal-place values of the approximate solution and actual solution at the points $x = 0.1, 0.2, 0.3, 0.4, 0.5$.

$$y' = -y \qquad y(x) = 2e^{-x}$$
$$y(0) = 2$$

$$x_0 = 0, \quad y_0 = 2 \qquad y(0) = 2$$

$$k_1 = f(0, 2) = -2$$
$$u_{n+1} = 2 + (.1)(-2) = 1.8$$
$$k_2 = f(.1, 1.8) = -1.8$$
$$y_1 = 2 + (.1) \cdot \tfrac{1}{2}(-2 + (-1.8))$$
$$= 1.81$$

$$x_1 = .1 \quad y_1 = 1.81 \quad y(.1) = 1.8097$$

$$k_1 = f(.1, 1.81) = -1.81$$
$$u_2 = 1.81 + (.1)(-1.81) = 1.629$$
$$k_2 = f(.2, 1.629) = -1.629$$
$$y_2 = 1.81 + (.1) \cdot \tfrac{1}{2}(-1.81 - 1.629)$$
$$= 1.6381$$

$x_2 = .2 \quad y_2 = 1.6381 \quad y(.2) = 1.6375$

$k_1 = f(.2, 1.6381) = -1.6381$
$u_3 = 1.6381 + .1(-1.6381) = 1.4742$
$k_2 = f(.3, 1.4742) = -1.4742$
$y_3 = 1.6381 + .1(.5)(-1.6381 - 1.4742)$
$= 1.4825$

$x_3 = .3 \quad y_3 = 1.4825 \quad y(.3) = 1.4816$

$k_1 = -1.4825$
$u_4 = 1.4825 + .1(-1.4825) = 1.3342$
$k_2 = -1.3342$
$y_4 = 1.4825 + .1(\frac{1}{2})(-1.4825 - 1.3342)$
$= 1.3417$

$x_4 = .4 \quad y_4 = 1.3417 \quad y(.4) = 1.3406$

$k_1 = -1.3417$
$u_5 = 1.3417 + .1(-1.3417) = 1.2075$
$k_2 = -1.2075$
$y_5 = 1.3417 + .1(\frac{1}{2})(-1.3417 - 1.2075)$
$= 1.2142$

$x_5 = .5 \quad y_5 = 1.2142 \quad y(.5) = 1.2131$

n	x_n	y_n	$y(x_n)$
0	0	2	2
1	.1	1.81	1.8097
2	.2	1.6381	1.6375
3	.3	1.4825	1.4816
4	.4	1.3417	1.3406
5	.5	1.2142	1.2131

2.5.5. $y' = y - x - 1 \quad y(0) = 1 \quad \cancel{y(x) = 2 + x + e}$

$y(x) = 2 + x - e^x$

$x_0 = 0 \quad y_0 = 1 \quad y(0) = 1$

$$\begin{aligned} k_1 &= 0 \\ u_1 &= 1 + (.1)(0) = 1 \\ k_2 &= f(.1, 1) = -.1 \\ y_1 &= 1 + .1(\tfrac{1}{2})(0 - .1) \\ &= .995 \end{aligned}$$

$x_1 = .1 \quad y_1 = .995 \quad y(.1) = .9948$

$$\begin{aligned} k_1 &= -.105 \\ u_2 &= .995 + (.1)(-.105) = .9845 \\ k_2 &= -.2155 \\ y_2 &= .995 + .1(\tfrac{1}{2})(-.105 - .2155) \\ &= .9740 \end{aligned}$$

$\frac{dy}{dx} = y - x - 1$

$x_2 = .2 \quad y_2 = .9790 \quad y(.2) = .9786$

$k_1 = -.221$
$u_3 = .9790 + .1(-.221) = .9569$
$k_2 = f(.3, .9569) = -.3431$
$y_3 = .9790 + .1(\frac{1}{2})(-.221 - .3431)$
$= .9508$

$x_3 = .3 \quad y_3 = .9508 \quad y(.3) = .9501$

$k_1 = -.3492$
$u_4 = .9159$
$k_2 = -.4841$
$y_4 = \cancel{.9159}\ .9508 + .1(\frac{1}{2})(-.3492 - .4841)$
$= \cancel{.8742}\ .9091$

$x_4 = .4 \quad \cancel{y_4 = .8742} \quad y(.4) = .9082$
$y_4 = .9091$
$\cancel{k_1 = -.5258}$
$\cancel{u_5 = .8216}$
$\cancel{k_2 = -.6784}$
$\cancel{y_5 = .8216 + .1(\frac{1}{2})(-.5258 - .6784)}$
$\cancel{= .7614}$

$\cancel{x_5 = .5 \quad y_5 = .7614 \quad y(.5) = .8513}$

$k_1 = -.4909$
$u_5 = .8601$
$k_2 = -.6399$
$y_5 = .8526$

$x_5 = .5 \quad y_5 = .8526 \quad y(.5) = .8513$

n	x_n	y_n	$y(x_n)$
1	.1	.995	.9948
2	.2	.9790	.9786
3	.3	.9508	.9501
4	.4	.9091	.9082
5	.5	.8526	.8913

2.5.9 $y_0 = 1 \quad x_0 = 0 \quad y' = \frac{1}{4}(1+y^2) \quad y = \tan\left(\frac{x+\pi}{4}\right)$

$k_1 = f(0,1) = 1/2$
$u_1 = 1 + .1(1/2) = 1.05$
$k_2 = f(.1, 1.05) = .5256$
$y_1 = 1 + .1(.5)(.5 + .5256)$
$= 1.0513$

$y_1 = 1.0513 \quad x_1 = .1 \quad y(.1) = 1.0513$

$k_1 = f(.1, 1.0513) = .5263$
$u_2 = 1.0513 + .1(.5263) = 1.1039$
$k_2 = f(.2, 1.1039) = .5546$
$y_2 = 1.0513 + .1(.5)(.5263 + .5546)$
$= 1.1053$

$y_2 = 1.1053 \quad x_2 = .2 \quad y(.2) = 1.1053$

$k_1 = f(.2, 1.1053) = .5554$
$u_3 = 1.1053 + .1(.5554) = 1.16084$
$k_2 = .5869$
$y_3 = 1.1624$

$y_3 = 1.1624 \quad x_3 = .3 \quad y(.3) = 1.1625$

$$k_1 = .5878$$
$$u_4 = 1.1624 + .1(.5878) = 1.2212$$
$$k_2 = .6228$$
$$y_4 = 1.1624 + .1\left(\tfrac{1}{2}\right)(.5878 + .6228)$$
$$= 1.2229$$

$y_4 = 1.2229 \quad x_4 = .4 \quad y(.4) = 1.2230$

$$k_1 = .6239$$
$$u_5 = 1.2893$$
$$k_2 = .6630$$
$$y_5 = 1.2872$$

$y_5 = 1.2872 \quad x_5 = .5 \quad y(.5) = 1.2874$

n	x_n	y_n	$y(x_n)$
0	0	1	1
1	.1	1.0513	1.0513
2	.2	1.1053	1.1053
3	.3	1.1624	1.1625
4	.4	1.2229	1.2230
5	.5	1.2872	1.2874

2.5.10 $y' = 2xy^2 \quad y(0)=1 \quad y(x) = \frac{1}{1-x^2}$

$y_0 = 1 \quad x_0 = 0 \quad y(0) = 1$

$k_1 = 0$
$u_1 = 1 + .1(0) = 1$
$k_2 = 2(.1)(1^2) = .2$
$y_1 = 1 + .1(\tfrac{1}{2})(0 + .2)$
$= 1.01$

$y_1 = 1.01 \quad x_1 = .1 \quad y(.1) = 1.0101$

$k_1 = .2040$
$u_2 = 1.01 + .1(.2040) = 1.0304$
$k_2 = .4247$
$y_2 = 1.01 + .1(\tfrac{1}{2})(.2040 + .4247)$
$= \boxed{1.0414}$

$y_2 = 1.0414 \quad x_2 = .2 \quad y(.2) = 1.0417$

$k_1 = .4338$
$u_3 = 1.0848$
$k_2 = .7061$
$y_3 = 1.0414 + .1(\tfrac{1}{2})(.4338 + .7061)$
$= 1.0984$

$y_3 = 1.0984 \quad x_3 = .3 \quad y(.3) = 1.0989$

$k_1 = .7239$
$u_4 = 1.0984 + .1(.7239) = 1.1708$
$k_2 = 1.0966$
$y_4 = 1.0984 + .1(\tfrac{1}{2})(.7239 + 1.0966)$
$= 1.1894$

$y_4 = 1.1894 \qquad x_4 = .4 \qquad y(.4) = 1.1905$

$k_1 = 1.1317$
$u_5 = 1.1894 + .1(1.1317) = 1.3026$
$k_2 = 1.6968$
$y_5 = 1.1894 + .1(\tfrac{1}{2})(1.1317 + 1.6968)$
$= 1.3308$

$y_5 = 1.3308 \qquad x_5 = .5 \qquad y(.5) = 1.3333$

n	x_n	y_n	$y(x_n)$
0	0	1	1
1	.1	1.01	1.0101
2	.2	1.0414	1.0417
3	.3	1.0984	1.0989
4	.4	1.1894	1.1905
5	.5	1.3308	1.3333

2.5.26 Suppose the deer population $P(t)$ of a small forest initially numbers 25 and satisfies the logistic equation

$$\frac{dP}{dt} = 0.0225P - 0.0003P^2$$

with t in months. Use the improved Euler's method with a programmable calculator to approximate the solution for 10 years, first with step size $h=1$ and then with $h=.5$, rounding off to 3 decimal places. What percentage of the limiting population of 75 deer has been attained after 5 years? After 10 years?

With $h=1$ I get:

~~10 ye~~

5 years $\approx$ 48 dear
10 years $\approx$ 61 dear

With $h=.5$ I get:

5 years $\approx$ 49.011 dear 65.3% of max
10 years $\approx$ 65.936 dear 87.9% of max.

2.6.1 Apply the Runge-Kutta method to approximate this solution on the interval [0, 0.5] with step size h = .25. Construct a table showing five-decimal-place values of the approximate solution and actual solution at the points x = 0.25 and x = 0.5

$y' = -y \qquad y(0) = 2 \qquad y(x) = 2e^{-x}$

$x_0 = 0 \qquad y_0 = 2$

$K_1 = -2$
$K_2 = -1.75$
$K_3 = -1.78125$
$K_4 = -1.55469$

$x_1 = .25 \qquad y_1 = 1.55762 \qquad y(.25) = 1.55760$

$K_1 = -1.55762$
$K_2 = -1.36292$
$K_3 = -1.38725$
$K_4 = -1.38421$

$x_2 = .5 \qquad y_2 = 1.20586 \qquad y(.5) = 1.21306$

2.6.4 $y' = x - y$; $y(0) = 1$; $y(x) = 2e^{-x} + x - 1$

$y_0 = 1 \qquad x_0 = 0$

$K_1 = -1$
$K_2 = -.75$
$K_3 = -.78125$
$K_4 = -.5546875$

$x_1 = .25 \quad y_1 = .80762 \quad y(.25) = .80760$

$K_1 = -.55762$
$K_2 = -.36292$
$K_3 = -.38726$
$K_4 = -.21081$

$x_2 = .5 \quad y_2 = .71309 \quad y(.5) = .71306$

2.6.7. $y' = -3x^2y \quad y(0)=3 \quad y(x) = 3e^{-x^3}$

$y_0 = 3 \qquad x_0 = 0$

$k_1 = 0$
$k_2 = -.14063$
$k_3 = -.13980$
$k_4 = -.55595$

$x_1 = .25 \quad y_1 = 2.95347 \quad y(.25) = 2.95349$

$k_1 = -.55378$
$k_2 = -1.21679$
$k_3 = -1.18183$
$k_4 = -1.99390$

$x_2 = .5 \quad y_2 = 2.64745 \quad y(.5) = 2.64749$

2.6.10 $y' = 2xy^2$; $y(0) = 1$; $y(x) = \frac{1}{1-x^2}$

$x_0 = 0$ $y_0 = 1$ $y(0) = 1$

$k_1 = 0$
$k_2 = .25$
$k_3 = .25781$
$k_4 = \cancel{.56445}\ .53223$

$x_1 = .25$ $y_1 = 1.06449$ $y(.25) = 1.06667$

$k_1 = .56657$
$k_2 = .89149$
$k_3 = .87820$
$k_4 = \cancel{1.28404}\ 1.64877$

$x_2 = .5$ $y_2 = \cancel{1.28774}$
$y_2 = 1.30094$ $y(.5) = 1.33333$

..6.26 As in problem 26 of section 2.5, suppose the deer population $P(t)$ in a small forest initially numbers 25 and satisfies the logistic equation

$$\frac{dP}{dt} = 0.0225P - 0.0003P^2$$

(with t in months). Use the Runge-Kutta method with a programmable calculator or computer to approximate the solution for 10 years, first with step size $h = 6$ and then with $h = 3$, rounding off approximate P-values to four decimal places. What percentage of the limiting population of 75 deer has been attained after 5 years? After 10 years?

I ~~programm~~ wrote a Maple program to do this.

With step size $h = 6$:
After 5 years we predict: 49 deer
65% of limiting population
After 10 years we predict: 66 deer
88% of limiting population.

With step size $h = 3$:
After 5 years we predict: 49 deer
65% of limiting population
After 10 years we predict: 66 deer
88% of limiting population.