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Math 2280

Assignment #10 Solutions

7.1.1. Find the Laplace transform of the function using the definition.

$$f(t) = 1$$

$$F(s) = \int_0^\infty e^{-st}\,dt$$

$$= \left.\frac{e^{-st}}{-s}\right|_0^\infty = 0 - \left(-\frac{1}{s}\right) = \boxed{\frac{1}{s}}$$

7.1.6. $f(t) = \sin^2(t)$

$$F(s) = \int_0^\infty e^{-st}\sin^2(t)\,dt$$

$$= \int_0^\infty e^{-st}\left(\frac{1-\cos(2t)}{2}\right)dt$$

$$= -\left.\frac{e^{-st}}{2s}\right|_0^\infty - \int_0^\infty \frac{e^{-st}\cos(2t)}{2}\,dt$$

$$= \frac{1}{2s} - \frac{1}{2}\int_0^\infty e^{-st}\cos(2t)\,dt$$

Now,

$$\int_0^\infty e^{-st}\cos(2t)\,dt \qquad u = e^{-st} \quad du = -se^{-st}dt \qquad dv = \cos(2t)dt \quad v = \sin(2t)/2$$

$$= \left.\frac{e^{-st}\sin(2t)}{2}\right|_0^\infty + \frac{s}{2}\int_0^\infty e^{-st}\sin(2t)$$

$$= \frac{s}{2}\int_0^\infty e^{-st}\sin(2t) = \left.\frac{-\cancel{s}e^{-st}\cos(2t)}{\cancel{2}4}\right|_0^\infty - \frac{s^2}{4}\int_0^\infty e^{-st}\cos(2t)dt \qquad u = e^{-st} \quad du = -se^{-st} \qquad dv = \sin(2t)dt \quad v = -\cos(2t)/2$$

$$\cancel{= \frac{1}{2} - \frac{s^2}{4}\int_0^\infty e^{-st}\cos(2t)}$$

~~So,~~

$$\cancel{\int_0^\infty e^{-st}\cos(2t)\,dt = \frac{1}{2} - \frac{s^2}{4}\int_0^\infty e^{-st}\cos(2t)\,dt}$$

$$\cancel{\Rightarrow \frac{4+s^2}{4}\int_0^\infty e^{-st}\cos(2t) = \frac{1}{2}}$$

$$\cancel{\Rightarrow \int_0^\infty e^{-st}\cos(2t) = \frac{2}{s^2+4}}$$

$$= \frac{s}{4} - \frac{s^2}{4}\int_0^\infty e^{-st}\cos(2t)$$

So,

$$\int_0^\infty e^{-st}\cos(2t)\,dt = \frac{s}{4} - \frac{s^2}{4}\int_0^\infty e^{-st}\cos(2t)$$

$$\Rightarrow \frac{4+s^2}{4}\int_0^\infty e^{-st}\cos(2t)\,dt = \frac{s}{4}$$

$$\Rightarrow \int_0^\infty e^{-st}\cos(2t)\,dt = \frac{s}{s^2+4}$$

So,

$$F(s) = \frac{1}{2s} - \frac{\frac{s}{2}}{s^2+4} = \boxed{\frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+4}\right)}$$

7.1.20 Find the Laplace transform:

$$f(t) = te^t$$

$$F(s) = \int_0^\infty e^{-st} t e^t \, dt$$

$$= \int_0^\infty t e^{(1-s)t} dt \qquad u = t \qquad du = dt$$

$$dv = e^{(1-s)t} dt \qquad v = \frac{e^{(1-s)t} dt}{(1-s)}$$

$s > 1$

$$= \frac{t e^{(1-s)t} dt}{(1-s)} \Big|_0^\infty - \int_0^\infty \frac{e^{(1-s)t} dt}{(1-s)}$$

$$= -\frac{1}{(1-s)} \int_0^\infty e^{-st} e^t dt$$

$$= \frac{1}{s-1} \mathcal{L}(e^t) = \boxed{\frac{1}{(s-1)^2}}$$

7.1.30. Find the inverse Laplace transform:

$$F(s) = \frac{9+s}{4-s^2} = -\frac{9}{s^2-4} - \frac{s}{s^2-4}$$

$$= -\frac{9}{2}\left(\frac{2}{s^2-4}\right) - \frac{s}{s^2-4}$$

Taking the inverse Laplace transform we get:

$$\cancel{\frac{9}{4}} \quad \boxed{-\frac{9}{2}\sinh(2t) - \cosh(2t)}$$

7.1.36. Show that the function $f(t) = \sin(e^{t^2})$ is of exponential order as $t \to \infty$ but that its derivative is not.

We note $|\sin(x)| \leq 1$ for any x. So, for $c = 0$, $M = 1$, and $T = 0$ we have

$$|\sin(e^{t^2})| \leq Me^{ct} \quad \text{for } t \geq T.$$

So, $f(t) = \sin(e^{t^2})$ is of exponential order

On the other hand

$$f'(t) = 2t\, e^{t^2} \cos(e^{t^2})$$

$$\lim_{t \to \infty} \frac{f'(t)}{e^{ct}} = \lim_{t \to \infty} 2t\, e^{t^2 - ct} \cos(e^{t^2})$$

Now, as $t \to \infty$ $e^{t^2 - ct} \to \infty$ for <u>any</u> $c > 0$. And, $\cos(e^{t^2})$ never settles down to 0, so we have that for <u>No</u> value of M, c, T is it the case

$$|2te^{t^2 - ct} \cos(e^{t^2})| \leq Me^{ct}$$

So, $f'(t)$ is not of exponential order.

7.2.1. Use Laplace transforms to solve the initial value problems:

$$x'' + 4x = 0 \quad ; \quad x(0) = 5, \quad x'(0) = 0$$

$$\mathcal{L}(x'' + 4x) = 0$$

$$\Rightarrow s^2 X(s) - 5s + 4X(s) = 0$$

$$\Rightarrow X(s) = \frac{5s}{s^2+4}$$

the inverse Laplace transform is:

$$x(t) = 5\cos(2t)$$

7.2.4 $$x'' + 8x' + 15x = 0 \quad ; \quad x(0) = 2, \quad x'(0) = -3$$

$$\mathcal{L}(x'' + 8x' + 15x) = 0$$

$$\Rightarrow s^2 X(s) - 2s + 3 + 8sX(s) - 16 + 15X(s) = 0$$

$$\Rightarrow (s^2+8s+15)X(s) = 2s + 13$$

$$\Rightarrow X(s) = \frac{2s+13}{s^2+8s+15} = \frac{2s+13}{(s+3)(s+5)}$$

So,

$$X(s) = \frac{A}{s+3} + \frac{B}{s+5}$$

$$\Rightarrow \begin{matrix} A + B = 2 \\ 5A + 3B = 13 \end{matrix} \quad \Rightarrow 2A = 7 \Rightarrow A = 7/2, \quad B = -3/2$$

So,

$$X(s) = \frac{7/2}{s+3} - \frac{3/2}{s+5}$$

The inverse Laplace transform is:

$$\boxed{x(t) = \frac{7}{2} e^{-3t} - \frac{3}{2} e^{-5t}}$$

7.2.15

$$x'' + x' + y' + 2x - y = 0 \qquad x(0) = y(0) = 1$$
$$y'' + x' + y' + 4x - 2y = 0 \qquad x'(0) = y'(0) = 0$$

Taking the Laplace transforms:

$$s^2 X(s) - s + sX(s) - 1 + sY(s) - 1 + 2X(s) - Y(s) = 0$$
$$s^2 Y(s) - s + sX(s) - 1 + sY(s) - 1 + 4X(s) - 2Y(s) = 0$$

$$\Rightarrow (s^2+s+2) X(s) + (s-1) Y(s) = s+2$$
$$(s+4) X(s) + (s^2+s-2) Y(s) = s+2$$

1. $(s^2+s+2) X(s) + (s-1) Y(s) = s+2$
2. $(s+4) X(s) + (s+2)(s-1) Y(s) = s+2$

$$Y(s) = \frac{s+2}{s-1} - \frac{(s^2+s+2)}{s-1} X(s) \quad \text{from equation 1.}$$

$$(s+4) X(s) + (s+2)(s-1)\left[\frac{s+2}{s-1} - \frac{(s^2+s+2)}{s-1} X(s)\right] = s+2 \quad \text{from equation 2}$$

$$(s+4) X(s) + s+2\left(s+2 - (s^2+s+2) X(s)\right) = s+2$$

$$\Rightarrow \quad \left[(s+4) - (s+2)(s^2+s+2) \right] X(s) = (s+2) - (s+2)^2$$

$$\left[s+4 - s^3 - s^2 - 2s - 2s^2 - 2s - 4 \right] X(s) = -s^2 - 3s - 2$$

$$\Rightarrow \quad (-s^3 - 3s^2 - 3s) X(s) = -s^2 - 3s - 2$$

$$\Rightarrow \quad X(s) = \frac{s^2+3s+2}{s(s^2+3s+3)} = \cancel{As} \frac{A}{s} + \frac{Bs+C}{s^2+3s+3}$$

$$\Rightarrow \quad \begin{array}{l} A + B = 1 \\ 3A + C = 3 \\ 3A = 2 \end{array} \quad \Rightarrow \quad \begin{array}{l} A = 2/3 \\ B = 1/3 \\ C = 1 \end{array}$$

$$\Rightarrow \quad X(s) = \frac{2/3}{s} + \frac{\frac{1}{3} s}{(s+\frac{3}{2})^2 + \frac{3}{4}} + \frac{1}{(s+\frac{3}{2})^2 + 3/4}$$

writing this in another way:

$$X(s) = \frac{2}{3}\left(\frac{1}{s}\right) + \frac{1}{3}\left(\frac{s+3/2}{(s+\frac{3}{2})^2 + 3/4}\right) + \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}/2}{(s+\frac{3}{2})^2 + 3/4}\right)$$

which has inverse Laplace transform:

$$x(t) = \frac{2}{3} + e^{-\frac{3}{2}t}\left[\frac{1}{3}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{1}{\sqrt{3}} \sin\left(\frac{\sqrt{3}}{2}t\right)\right]$$

$$\cancel{= \frac{1}{9}\left(6\right.}$$

$$= \boxed{\frac{1}{3}\left(2 + e^{-3/2\, t}\left(\cos\left(\frac{\sqrt{3}}{2}t\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)\right)}$$

Now, to solve for $Y(s)$:

$$(s^2+s+2)X(s) + (s-1)Y(s) = s+2$$
$$(s+4)X(s) + (s+2)(s-1)Y(s) = s+2$$

$$\Rightarrow X(s) = \frac{s+2-(s+2)(s-1)Y(s)}{s+4}$$

$$\Rightarrow (s^2+s+2)\left(\frac{(s+2)-(s+2)(s-1)Y(s)}{s+4}\right) + (s-1)Y(s) = s+2$$

$$\Rightarrow (s^2+s+2)\left((s+2)-(s+2)(s-1)Y(s)\right) + (s+4)(s-1)Y(s) = (s+2)(s+4)$$

$$Y(s) = \frac{(s^2-2)(s+2)}{(s^2+s+2)(s+2)(s-1)-(s+4)(s-1)}$$

$$= \frac{s^3+2s^2-2s-4}{s^4+2s^3-3s}$$

$$= \frac{s^3+2s^2-2s-4}{s(s-1)(s^2+3s+3)}$$

$$= \frac{A}{s} + \frac{B}{s-1} + \frac{Cs+D}{s^2+3s+3}$$

$$\begin{aligned} \Rightarrow A + B + C \quad &= 1 \\ 2A + 3B - C + D &= 2 \\ 3B \quad - D &= -2 \\ -3A \quad &= -4 \end{aligned}$$

$$\Rightarrow A = 4/3$$

$$\Rightarrow \quad \begin{aligned} B + C \quad &= -1/3 \\ 3B - C + D &= -2/3 \\ 3B \quad - D &= -2 \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} B + C &= -1/3 \\ 6B - C &= -8/3 \end{aligned}$$

$$\Rightarrow \quad 7B = -3$$

$$\Rightarrow \quad B = -3/7$$

So,

$$C = -1/3 + 3/7 = -7/21 + 9/21 = 2/21$$

$$D = 3B + 2 = -9/7 + 14/7 = 5/7$$

So,

$$A = 28/21 \quad B = -9/21$$
$$C = 2/21 \quad D = 15/21$$

Therefore,

$$Y(s) = \frac{1}{21}\left(\frac{28}{s} - \frac{9}{s-1} + \frac{2s+15}{(s+\frac{3}{2})^2+\frac{3}{4}}\right)$$

$$= \frac{1}{21}\left(\frac{28}{s} - \frac{9}{s-1} + 2\left(\frac{s+\frac{3}{2}}{(s+\frac{3}{2})^2+\frac{3}{4}}\right) + \frac{24}{\sqrt{3}}\left(\frac{\frac{\sqrt{3}}{2}}{(s+\frac{3}{2})^2+\frac{3}{4}}\right)\right)$$

and the inverse Laplace transform is:

$$\boxed{y(t) = \frac{1}{21}\left(28 - 9e^{t} + 2e^{-3/2 t}\left(\cos\left(\frac{\sqrt{3}}{2}t\right) + 4\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)\right)}$$

7.2.20 Find the inverse Laplace transform of the functions:

$$F(s) = \frac{2s+1}{s(s^2+9)}$$

$$\cancel{= \frac{2}{s^2+9} + \frac{1}{s(s^2+9)}}$$

$$\cancel{= \frac{2}{s^2+9} + \frac{A}{s} +}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+9}$$

$$\Rightarrow \quad A(s^2+9) + Bs^2 + Cs = 2s+1$$

$$\begin{aligned} 9A &= 1 \\ C &= 2 \\ A+B &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} A &= 1/9 \\ B &= -1/9 \\ C &= 2 \end{aligned}$$

$$F(s) = \frac{1/9}{s} \cancel{+} -\frac{1}{9}\left(\frac{s-18}{s^2+9}\right)$$

$$= \frac{1}{9}\left(\frac{1}{s}\right) - \frac{1}{9}\left(\frac{s}{s^2+9}\right) + \frac{2}{3}\left(\frac{3}{s^2+9}\right)$$

So,

$$\boxed{f(t) = 1/9 - 1/9\cos(3t) + 2/3\sin(3t)}$$

7.2.29 Derive the Laplace transform by applying theorem 1 from 7.2.

$$\mathcal{L}\{t\sinh(kt)\}$$

$f(t) = t\sinh(kt) \qquad f(0) = 0$

$f'(t) = \sinh(kt) + kt\cosh(kt) \qquad f'(0) = 0$

$f''(t) = 2k\cosh(kt) + k^2 t\sinh(kt)$

Now,

$$\mathcal{L}(f''(t)) = s^2\mathcal{L}(f(t))$$

So,

$$2k\mathcal{L}\{\cosh(kt)\} + k^2\mathcal{L}\{t\sinh(kt)\} = s^2\mathcal{L}(t\sinh(kt)\}$$

Now, $\mathcal{L}\{\cosh(kt)\} = \dfrac{s}{s^2-k^2}$

$$\Rightarrow \mathcal{L}\{t\sinh(kt)\} = \boxed{\frac{2ks}{(s^2-k^2)^2}}$$

7.3.3. Apply the translation theorem to find the Laplace transform of the function.

$$f(t) = e^{-2t}\sin(3\pi t)$$

Now, the translation theorem tells us:

$$\mathcal{L}\{e^{-2t}\sin(3\pi t)\} = F(s+2)$$

where $F(s) = \mathcal{L}(\sin(3\pi t)) = \dfrac{3\pi}{s^2+9\pi^2}$

$$\Rightarrow F(s+2) = \boxed{\dfrac{3\pi}{(s+2)^2+9\pi^2}}$$

7.3.8. Find the inverse Laplace transform of the function:

$$F(s) = \frac{s+2}{s^2+4s+5} = \frac{s+2}{(s+2)^2+1}$$

which has inverse Laplace transform (using the table on page 466)

$$\boxed{f(t) = e^{-2t}\cos(t)}$$

7.3.19 Find the inverse Laplace transform:

$$F(s) = \frac{s^2 - 2s}{s^4 + 5s^2 + 4}$$

$$= \frac{s^2 - 2s}{(s^2+4)(s^2+1)}$$

$$= \frac{As + B}{s^2+4} + \frac{Cs + D}{s^2+1}$$

$$\Rightarrow \begin{aligned} A + C &= 0 \\ B + D &= 1 \\ A + 4C &= -2 \\ B + 4D &= 0 \end{aligned} \qquad \Rightarrow 3C = -2 \Rightarrow C = -2/3, \quad A = 2/3$$

$$3D = -1 \Rightarrow D = -1/3 \Rightarrow B = 4/3.$$

So,

$$= \frac{1}{3}\left(\frac{2s+4}{s^2+4}\right) + \frac{1}{3}\left(\frac{-2s-1}{s^2+1}\right)$$

$$= \frac{1}{3}\left(\frac{2s+4}{s^2+4}\right) - \frac{1}{3}\left(\frac{2s+1}{s^2+1}\right)$$

$$= \frac{2}{3}\left(\frac{s}{s^2+4}\right) + \frac{2}{3}\left(\frac{2}{s^2+4}\right) - \frac{2}{3}\left(\frac{s}{s^2+1}\right) - \frac{1}{3}\left(\frac{1}{s^2+1}\right)$$

which has inverse Laplace transform:

$$= \boxed{\frac{2}{3}\cos(2t) + \frac{2}{3}\sin(2t) - \frac{2}{3}\cos(t) - \frac{1}{3}\sin(t)}$$

7.3.24 Use the factorization:

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

to derive the inverse Laplace transform:

$$\mathcal{L}^{-1}\left\{ \frac{s}{s^4 + 4a^4} \right\} = \frac{1}{2a^2} \sinh(at)\sin(at)$$

$$\frac{s}{s^4 + 4a^4} = \frac{As + B}{s^2 - 2as + 2a^2} + \frac{Cs + D}{s^2 + 2as + 2a^2}$$

$\cancel{\Rightarrow A + C =}$

$$= (A+C)s^3 + (2aA + B - 2aC + D)s^2 + (2a^2A + 2aB \cancel{- 2aC + 2D})s + (2a^2B + 2a^2D)$$

$\cancel{+2a}$

$$= s$$

$$\begin{aligned} A \quad + C &= 0 \\ 2aA + B - 2aC + D &= 0 \\ 2a^2A + 2aB - 2aC + & \end{aligned}$$

$$= \frac{(A+C)s^3 + (2aA + B - 2aC + D)s^2 + (2a^2A + 2aB + 2a^2C - 2aD)s + (2a^2B + 2a^2D)}{s^4 + 4a^4}$$

$$\begin{aligned} \Rightarrow \quad A \qquad + C \qquad &= 0 \\ 2aA + B - 2aC + D &= 0 \\ 2a^2A + 2aB + 2a^2C - 2aD &= 1 \\ 2a^2B \qquad + 2a^2D &= 0 \end{aligned}$$

So, $A=-C$ from the first equality. Plugging this into the third ~~way~~ we get:

$$2aB - 2aD = 1$$

while from the fourth equality we have $B=-D$. So,

$$4aB = 1 \Rightarrow B = 1/4a$$

while

$$D = -1/4a$$

The second equality tells us, given $B=-D$, that $A=C$. As $A=-C$ this implies $A=C=0$.

So, we get the decomposition:

$$\frac{s}{s^4+4a^4} = \frac{1}{4a}\left(\frac{1}{s^2-2as+2a^2} - \frac{1}{s^2+2as+2a^2}\right)$$

$$= \frac{1}{4a}\left(\frac{1}{(s-a)^2+a^2} - \frac{1}{(s+a)^2+a^2}\right)$$

Taking the inverse Laplace transform:

$$= \frac{1}{4a}\left(\frac{e^{at}\sin(at)}{a} - \frac{e^{-at}\sin(at)}{a}\right)$$

$$= \frac{1}{2a^2}\left(\frac{e^{at}-e^{-at}}{2}\right)\sin(at)$$

$$= \boxed{\frac{1}{2a^2}\sinh(at)\sin(at)}$$

7.3.30 Use Laplace transforms to solve the following initial value problems:

$$x'' + 4x' + 8x = e^{-t} \qquad x(0) = x'(0) = 0.$$

taking the Laplace transform we get:

$$s^2 X(s) - s\,x(0) - x'(0) + 4s\,X(s) - 4x(0) + 8X(s)$$

$$= \frac{1}{s+1}$$

$$\Rightarrow \quad (s^2 + 4s + 8)\,X(s) = \frac{1}{s+1}$$

$$\Rightarrow \quad X(s) = \frac{1}{(s+1)\,[\,(s+2)^2 + 4\,]}$$

the partial fraction decomposition will be:

$$X(s) = \frac{A}{s+1} + \frac{Bs + C}{(s+2)^2 + 4}$$

$$= \frac{(A+B)s^2 + (4A + B + C)s + (8A + C)}{(s+1)\,[\,(s+2)^2 + 4\,]}$$

So,

$$A + B = 0 \qquad \Rightarrow A = -B$$

$$4A + B + C = 0$$

$$8A + C = \underline{1} \qquad \Rightarrow \begin{array}{l} 3A + C = 0 \\ 8A + C = 1 \end{array}$$

$$\Rightarrow A = 1/5, \quad B = -1/5, \quad C = -3/5$$

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$$X(s) = \frac{1}{5}\left(\frac{1}{s+1}\right) - \frac{1}{5}\left(\frac{s+2}{(s+2)^2+4}\right) \cancel{+} - \frac{\cancel{2}1}{10}\left(\frac{2}{(s+2)^2+4}\right)$$

taking the inverse Laplace transform we get:

$$\boxed{x(t) = \frac{e^{-t}}{5} - \frac{e^{-2t}\cos(2t)}{5} - \frac{e^{-2t}\sin(2t)}{10}}$$

7.3.33 $x^{(4)} + x = 0$; $x(0) = x'(0) = x''(0) = 0$, $x^{(3)}(0) = 1$

taking the Laplace transform we get:

$$s^4 X(s) - s^3 x(0) - s^2 x'(0) - s x''(0) - x^{(3)}(0) + X(s) = 0$$

$$\Rightarrow (s^4+1)X(s) - 1 = 0$$

$$\Rightarrow X(s) = \frac{1}{s^4+1} = \cancel{\frac{1}{(s^2+1)(s^2-1)}}$$

$$\Rightarrow X(s) = \frac{1}{(s^2+\sqrt{2}s+1)(s^2-\sqrt{2}s+1)}$$

$$= \frac{As+B}{(s+\frac{1}{\sqrt{2}})^2+\frac{1}{2}} + \frac{Cs+D}{(s-1/\sqrt{2})^2+1/2}$$

$$= (A+C)s^3 + (-\sqrt{2}A+B+\sqrt{2}C+D)s^2 + (A-\sqrt{2}B+C+\sqrt{2}D)s + (B+D) = 1$$

So,

$$\begin{aligned}
A \qquad\quad + C \qquad &= 0 \\
-\sqrt{2}A \quad + B + \sqrt{2}C + D &= 0 \\
A - \sqrt{2}B + C + \sqrt{2}D &= 0 \\
B \qquad + D &= 1
\end{aligned}$$

So, $A = -C$, and therefore

$$\begin{aligned}
-\sqrt{2}\,B + \sqrt{2}\,D &= 0 \qquad \Rightarrow \qquad -B + D = 0 \\
B + D &= 1 \qquad\qquad\qquad\;\; B + D = 1
\end{aligned}$$

$$\Rightarrow \quad D = 1/2, \quad B = 1/2.$$

and,

$$\begin{aligned}
A + C &= 0 \\
-\sqrt{2}A + \sqrt{2}C &= -1
\end{aligned}$$

$$\Rightarrow \quad 2\sqrt{2}\,C = -1 \quad \Rightarrow \quad C = -\sqrt{2}/4 \qquad A = \sqrt{2}/4.$$

So,

$$\frac{1}{s^2+4} = \frac{1}{4}\left(\frac{\sqrt{2}s+2}{(s+1/\sqrt{2})^2+1/2}\right) \cancel{+} - \frac{1}{4}\left(\frac{\sqrt{2}s-2}{(s-1/\sqrt{2})^2+1/2}\right)$$

$$\cancel{= \frac{\sqrt{2}}{4}\left(\frac{s}{(s+1/\sqrt{2})^2+1/2}\right) + \frac{\sqrt{2}}{2}\left(\frac{\frac{1}{\sqrt{2}}}{(s+1/\sqrt{2})^2+1/2}\right)}$$

$$-$$

If we simplify this we get:

$$= \frac{\sqrt{2}}{4}\left(\frac{s+1/\sqrt{2}}{(s+1/\sqrt{2})^2+1/2}\right) + \frac{\sqrt{2}}{4}\left(\frac{1/\sqrt{2}}{(s+1/\sqrt{2})^2+1/2}\right)$$

$$- \frac{\sqrt{2}}{4}\left(\frac{s-1/\sqrt{2}}{(s-1/\sqrt{2})^2+1/2}\right) + \frac{\sqrt{2}}{4}\left(\frac{1/\sqrt{2}}{(s+1/\sqrt{2})^2+1/2}\right)$$

which has inverse Laplace transform:

$$\boxed{\begin{aligned} x(t) = &\frac{\sqrt{2}}{4} e^{-t/\sqrt{2}} \cos(t/\sqrt{2}) + \frac{\sqrt{2}}{4} e^{-t/\sqrt{2}} \sin(t/\sqrt{2}) \\ &- \frac{\sqrt{2}}{4} e^{t/\sqrt{2}} \cos(t/\sqrt{2}) + \frac{\sqrt{2}}{4} e^{t/\sqrt{2}} \sin(t/\sqrt{2}) \end{aligned}}$$

we note that we can group these as:

$$\frac{\sqrt{2}}{2}\left(\frac{e^{-t/\sqrt{2}} - e^{t/\sqrt{2}}}{2} \cos(t/\sqrt{2}) + \frac{e^{-t/\sqrt{2}} + e^{t/\sqrt{2}}}{2} \sin(t/\sqrt{2})\right)$$

$$= \frac{1}{\sqrt{2}}\left(\cosh(t/\sqrt{2})\sin(t/\sqrt{2}) - \sinh(t/\sqrt{2})\cos(t/\sqrt{2})\right)$$

which is the form in the back of the book.