Math 2280 - Quiz 3

Instructor: Dylan Zwick

Spring 2008

Name: ____

50 Points Possible

Note - For credit you must show your work on all of these problems. A solution, even a correct solution, with no work or essentially no work will receive very little credit.

1. Calculate the following: (9 points)

   a) Using the formal definition of the Laplace transform (i.e. calculate the integral) what is the Laplace transform of the function:

   $$f(t) = t - 2e^{3t}$$

   also state the domain of the Laplace transform. (3 points)

b) Calculate the convolution $f(t) * g(t)$ of the functions:

$$f(t) = t^2 \text{ and } f(t) = \cos t$$

(3 points)

c) Again using the formal definition calculate the Laplace transform:

$$f(t) = t^2$$

again state the domain of the Laplace transform. (3 points)

2. Solve the initial value problem for the function $x(t)$:

$$x'' + 4x' + 5x = \delta(t - \pi) + \delta(t - 2\pi);$$
$$x(0) = 0,\ x'(0) = 2.$$

(8 points)

Continued

3. Determine if $x = 0$ is an ordinary, regular singular, or irregular singular point in each of the following differential equations: (9 points)

a) (3 points)

$$3x^3y'' + 2x^2y' + (1 - x^2)y = 0$$

b) (3 points)

$$x^2(1-x^2)y'' + 2xy' - 2y = 0$$

c) (3 points)

$$xy'' + x^2y' + (e^x - 1)y = 0$$

4. Solve the following second-order ODE using power series or Frobenius series methods:

$$y'' + x^2 y' + 2xy = 0$$

(8 points)

Continued

5. Solve the following second-order ODE using power series or Frobenius series methods:

$$2xy'' - y' - y = 0$$

(8 points)

Continued

6. Find the odd extension of the function defined below, and graph the odd extension. Then, calculate the corresponding Fourier series (sine series) representation of the odd extension.

$$f(t) = t^2, 0 < t < \pi.$$

(8 points)

Continued

7. (Extra Credit) Derive the following equivalence:

$$\gamma(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}$$

where the product on the right is over prime numbers p. In your derivation you can use the relation:

$$\sum_{k=0}^{\infty} \frac{1}{p^{ks}} = \left(1 - \frac{1}{p^s}\right)^{-1}.$$

(5 points)

Continued