Math 2280 - Final Exam Part 2

Instructor: Dylan Zwick

Spring 2008

Name: ______________________________

50 Points Possible

Note - This is the half of the exam that is open book. You may look at this half of the exam, and even start it if you'd like, before you hand in the first half. However, before you open the book to work on this half you must hand in the first half.

1. Find the general solution to the following ODE: (10 points)

$$2xy^2 + (2x^2y + 4y^3)y' = -3x^2$$

2. Find the general solution to the following ODE: (10 points)

$$y'' + 9y = 2x^2 e^{3x} + 5$$

Continued...

3. Find the general solution to the system of ODEs described by the following matrix equation: (10 points)

$$\mathbf{x}' = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} \mathbf{x}$$

Continued...

4. Use Laplace transforms to solve the ODE: (10 points)

$$x'' + 2x + x = \delta(t) - \delta(t-2)$$
$$x(0) = x'(0) = 2$$

Continued..

5. Use either power series or Frobenius series methods to construct two linearly independent solutions to the differential equation: (10 points)

$$2xy'' + (1 - 2x^2)y' - 4xy = 0$$

Continued...

6. *Extra Credit* In this problem we will step through a derivation of a closed form equation for the nth term of the Fibonacci sequence. (10 points)

   a) The Fibonacci sequence is defined by the recurrence relation:

   $$x_{n+2} = x_{n+1} + x_n$$

   with $x_0 = 0$ and $x_1 = 1$.

   Suppose that we have a solution in the form $x_n = r^n$. If this "guess" is correct what must the value of r be for it to work? (There will be 2 possible values). (3 points)

b) In fact, any linear combination of the above two possible values of r will also satisfy the Fibonacci relation in the form $x_n = Ar_1^n + Br_2^n$. Find out what linear combination we need to use in order for it to square with our initial conditions, and from this derive a closed form solution for x_n. (4 points)

c) Define $y_n = \dfrac{x_{n+1}}{x_n}$. Figure out $\lim_{n\to\infty} y_n$. (2 points)

d) What famous number is this? (1 point)