HARTSHORNE'S ALGEBRAIC GEOMETRY - SECTION 2.1

Y.P. LEE'S CLASS

2.1.1: *Let A be an abelian group, and define the **constant presheaf** associated to A on the topological space X to be the presheaf $U \mapsto A$ for all $U \neq \emptyset$, with restriction maps the identity. Show that the constant sheaf $\mathcal{A}$ defined in the text is the sheaf associated to this presheaf.*

Solution by Dylan Zwick

If we examine the constant sheaf $\mathcal{A}$ we note that for an open set $U \subseteq X$ and a continuous map $\phi : U \to A$, with A given the discrete topology, there is an obvious and natural identification between $\phi(s)$, with $s \in U$, and the corresponding element in $\mathcal{F}_s$, where $\mathcal{F}_s$ is the stalk of the constant presheaf associated to A on X. Also, we note that for any $s \in U$ there is an open set $V \subseteq U$ defined by $\phi^{-1}(\phi(s))$[1], and an element $\phi(s) \in A = \mathcal{F}(V)$, such that for all $q \in V$ we have that $\phi(s)_q = \phi(q)$, where we again use the natural identification between $\phi(s)$ and the corresponding element in $\mathcal{F}_s$. So, the constant sheaf defined in the text is the sheafification of the constant presheaf defined here.

[1] We've used the discrete topology on A and the fact that ϕ is continuous