Math 2280 - Exam 3

University of Utah

Fall 2013

Name: ______________________________________

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

Things You Might Want to Know

Definitions

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)dt.$$

$$f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)d\tau.$$

Laplace Transforms

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a}$$

$$\mathcal{L}(\sin(kt)) = \frac{k}{s^2+k^2}$$

$$\mathcal{L}(\cos(kt)) = \frac{s}{s^2+k^2}$$

$$\mathcal{L}(\delta(t-a)) = e^{-as}$$

$$\mathcal{L}(u(t-a)f(t-a)) = e^{-as}F(s).$$

Translation Formula

$$\mathcal{L}(e^{at}f(t)) = F(s-a).$$

Derivative Formula

$$\mathcal{L}(x^{(n)}) = s^n X(s) - s^{n-1}x(0) - s^{n-2}x'(0) - \cdots - sx^{(n-2)}(0) - x^{(n-1)}(0).$$

1. (15 Points) *Calculating a Laplace Transform*

   Calculate the Laplace transform of the function

   $$f(t) = t - 4$$

   using the formal definition.

2. (15 Points) *Convolutions*

   Calculate the the convolution $f(t) * g(t)$ of the following functions:

$$f(t) = t \quad g(t) = \cos{(t)}.$$

3. (30 Points) *Delta Functions and Laplace Transforms*

Solve the initial value problem

$$x'' + 4x' + 4x = 1 + \delta(t-2).$$

$$x(0) = x'(0) = 0.$$

More room for Problem 3.

4. (10 Points) *Singular Points*

   Determine whether the point $x = 0$ is an ordinary point, a regular singular point, or an irregular singular point of the differential equation:

$$x^2(1 - x^2)y'' + 2xy' - 2y = 0.$$

5. (30 points) *Power Series*

   Use power series methods to find the general solution to the differential equation:

$$(x^2 + 2)y'' + 4xy' + 2y = 0.$$

   State the recurrence relation and the guaranteed radius of convergence.

More room for problem 5.