Math 2280 - Exam 2

University of Utah

Fall 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (10 Points) Taking the Lower (Order) Road

Convert the differential equation

$$x^{(4)} - 4x^{(3)} + 2x'' + 5x' - 7x = \cos 2t$$

into an equivalent system of first-order equations.

2. (15 Points) Take a Wild Guess

Use the method of undetermined coefficients to find the form of the particular solution to the differential equation:

$$y'' - 4y' + 3y = x^2 \cos(2x)e^{2x}$$

3. (25 Points) Linear Differential Equations

Find the solution to the initial value problem

$$x^{(3)} - 2x'' + x' = 0$$

with initial conditions x(0) = 3, x'(0) = 2, x''(0) = 3.

4. (25 Points) Solving First-Order Systems

Find the general solution to the series of first-order differential equations¹

$$\begin{aligned} x_1' &= x_1 + 2x_2 + 2x_3, \\ x_2' &= 2x_1 + 7x_2 + x_3, \\ x_3' &= 2x_1 + x_2 + 7x_3. \end{aligned}$$

¹Hint - All the eigenvalues are integers. And one of them is zero. If this is not what you're getting, check your algebra.

More room for this problem, if you need it.

5. (25 points) *Multiple Eigenvalues*²

Find the general solution to the system of differential equations given by the matrix equation:

$$\mathbf{x}' = \left(\begin{array}{cc} 7 & 1\\ -4 & 3 \end{array}\right) \mathbf{x}.$$

²The title of this problem is a hint.

More room for this problem, if you need it.