Math 2280 - Exam 1

University of Utah

Fall 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (20 Points) *Differential Equation Basics*

   (a) (5 points) What is the order of the differential equation given below?[1]

$$x^5 y^{(4)} + (e^{x^2} + 7x^3) y^{(3)} - \sin\left(y^{(5)}\right) + x^2 y' = y + x^2 - 2x + 7y^{(2)}$$

   (b) (5 points) Is the differential equation given below linear?

$$x^2 y^{(3)} - 2xy' + e^x = \sin(x) y''$$

   (c) (10 points) On what intervals are we guaranteed a unique solution exists for the differential equation below?

$$y' + e^x y = \frac{x+2}{x-1}$$

[1] Extra credit - Solve this differential equation! Just kidding. Do not attempt to solve it.

2. (10 points) *Phase Diagrams*

   Find the critical points for the autonomous equation:

$$\frac{dP}{dt} = kP(M - P)(P - H),$$

   where $k, M, H > 0$ and $M > H$. Draw the corresponding phase diagram, and indicate if the critical points are stable, unstable, or semistable.

3. (20 Points) *Separable Equations*

   Find the solution to the initial value problem:

$$\frac{dy}{dx} = 2xe^{x^2-y}$$

$$y(0) = 0.$$

4. (15 points) *Exact Equations*

   Find the solution to the initial value problem[2]:

$$\frac{dy}{dx} = -\frac{\cos(x) + ye^x}{e^x + 2y},$$

$$y(0) = 2.$$

[2]The title of this problem is a hint.

5. (20 points) *First-Order Linear Equations*

   Find a solution to the initial value problem given below, and give the interval upon which you know the solution is unique.

$$y' + 2xy = 3e^{-x^2}$$

$$y(0) = 4.$$

6. (15 points) *Euler's Method*

   Use Euler's method with step size $h = 1$ to estimate the solution to the initial value problem

$$\frac{dy}{dx} = x^2 + 2y - 1$$

$$y(0) = 3$$

   at $x = 2$.