Math 2280 - Assignment 4

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Section 3.1 - 1, 16, 18, 24, 30

Section 3.2 - 1, 10, 16, 24, 31

Section 3.3 - 1, 10, 25, 30, 43

Section 3.1 - Second-Order Linear Equations

3.1.1 Verify that the functions y_1 and y_2 given below are solutions to the second-order ODE also given below. Then, find a particular solution of the form $y = c_1y_1 + c_2y_2$ that satisfies the given initial conditions. Primes denote derivatives with respect to x.

$$y'' - y = 0;$$

$$y_1 = e^x \quad y_2 = e^{-x};$$

$$y(0) = 0 \quad y'(0) = 5.$$

3.1.16 Verify that the functions y_1 and y_2 given below are solutions to the second-order ODE also given below. Then, find a particular solution of the form $y = c_1 y_1 + c_2 y_2$ that satisfies the given initial conditions. Primes denote derivatives with respect to x.

$$x^2 y'' + xy' + y = 0;$$

$$y_1 = \cos(\ln x), \quad y_2 = \sin(\ln x);$$

$$y(1) = 2, \quad y'(1) = 3.$$

3.1.18 Show that $y = x^3$ is a solution of $yy'' = 6x^4$, but that if $c^2 \neq 1$, then $y = cx^3$ is not a solution.

3.1.24 Determine if the functions

$$f(x) = \sin^2 x, \quad g(x) = 1 - \cos(2x)$$

are linearly dependent on the real line $\mathbb{R}$.

3.1.30 **(a)** Show that $y_1 = x^3$ and $y_2 = |x^3|$ are linearly independent solutions on the real line of the equation $x^2y'' - 3xy' + 3y = 0$.

(b) Verify that $W(y_1, y_2)$ is identically zero. Why do these facts not contradict Theorem 3 from the textbook?

More room for Problem 3.1.30.

Section 3.2 - General Solutions of Linear Equations

3.2.1 Show directly that the given functions are linearly dependent on the real line. That is, find a non-trivial linear combination of the given functions that vanishes identically.

$$f(x) = 2x, \quad g(x) = 3x^2, \quad h(x) = 5x - 8x^2.$$

3.2.10 Use the Wronskian to prove that the given functions are linearly independent.

$$f(x) = e^x, \quad g(x) = x^{-2}, \quad h(x) = x^{-2}\ln x; \, x > 0.$$

3.2.16 Find a particular solution to the third-order homogeneous linear equation given below, using the three linearly independent solutions given below.

$$y^{(3)} - 5y'' + 8y' - 4y = 0;$$

$$y(0) = 1, \quad y'(0) = 4, \quad y''(0) = 0;$$

$$y_1 = e^x, \quad y_2 = e^{2x}, \quad y_3 = xe^{2x}.$$

3.2.24 Find a solution satisfying the given initial conditions for the differential equation below. A complementary solution y_c, and a particular solution y_p are given.

$$y'' - 2y' + 2y = 2x;$$

$$y(0) = 4 \quad y'(0) = 8;$$

$$y_c = c_1 e^x \cos x + c_2 e^x \sin x \quad y_p = x + 1.$$

3.2.31 This problem indicates why we can impose *only* n initial conditions on a solution of an nth-order linear differential equation.

(a) Given the equation

$$y'' + py' + qy = 0,$$

explain why the value of $y''(a)$ is determined by the values of $y(a)$ and $y'(a)$.

(b) Prove that the equation

$$y'' - 2y' - 5y = 0$$

has a solution satisfying the conditions

$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = C,$$

if and only if $C = 5$.

More room for problem 3.2.31.

Section 3.3 - Homogeneous Equations with Constant Coefficients

3.3.1 - Find the general solution to the differential equation

$$y'' - 4y = 0.$$

3.3.10 - Find the general solution to the differential equation

$$5y^{(4)} + 3y^{(3)} = 0.$$

3.3.25 - Solve the initial value problem

$$3y^{(3)} + 2y'' = 0;$$

$$y(0) = -1, \quad y'(0) = 0, \quad y''(0) = 1.$$

3.3.30 - Find the general solution to the differential equation

$$y^{(4)} - y^{(3)} + y'' - 3y' - 6y = 0.$$

3.3.43 -

(a) - Use Euler's formula to show that every complex number can be written in the form $re^{i\theta}$, where $r \geq 0$ and $-\pi < \theta \leq \pi$.

(b) - Express the numbers 4, -2, $3i$, $1+i$, and $-1+i\sqrt{3}$ in the form $re^{i\theta}$.

(c) - The two square roots of $re^{i\theta}$ are $\pm\sqrt{r}e^{i\theta/2}$. Find the square roots of the numbers $2-2i\sqrt{3}$ and $-2+2i\sqrt{3}$.

More room, if necessary, for Problem 3.3.43.