Math 2280 - Assignment 3

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Fall 2013

Section 2.3 - 1, 2, 4, 10, 24

Section 2.4 - 1, 5, 9, 26, 30

Section 2.3 - Acceleration-Velocity Models

2.3.1 The acceleration of a Maserati is proportional to the difference between 250 km/h and the velocity of this sports car. If the machine can accelerate from rest to 100 km/h in 10s, how long will it take for the car to accelerate from rest to 200 km/h?

More space, if necessary, for problem 2.3.1.

2.3.2 Suppose that a body moves through a resisting medium with resistance proportional to its velocity v, so that $dv/dt = -kv$.

(a) Show that its velocity and position at time t are given by

$$v(t) = v_0 e^{-kt}$$

and

$$x(t) = x_0 + \left(\frac{v_0}{k}\right)(1 - e^{-kt}).$$

(b) Conclude that the body travels only a finite distance, and find that distance.

More space, if necessary, for problem 2.3.2.

2.3.4 Consider a body that moves horizontally through a medium whose resistance is proportional to the *square* of the velocity v, so that

$$dv/dt = -kv^2.$$

Show that

$$v(t) = \frac{v_0}{1 + v_0 kt}$$

and that

$$x(t) = x_0 + \frac{1}{k} \ln \left(1 + v_0 kt\right).$$

Note that, in contrast with the result of Problem 2, $x(t) \to \infty$ as $t \to \infty$. Which offers less resistance when the body is moving fairly slowly - the medium in this problem or the one in Problem 2? Does your answer seem to be consistent with the observed behaviors of $x(t)$ as $t \to \infty$?

More space, if necessary, for problem 2.3.4.

2.3.10 A woman bails out of an airplane at an altitude of 10,000 ft, falls freely for 20s, then opens her parachute. How long will it take her to reach the ground? Assume linear air resistance ρv ft/s^2, taking $\rho = .15$ without the parachute and $\rho = 1.5$ with the parachute. (*Suggestion*: First determine her height above the ground and velocity when the parachute opens.)

More space, if necessary, for problem 2.3.10.

2.3.24 The mass of the sun is 329,320 times that of the earth and its radius is 109 times the radius of the earth.

(a) To what radius (in meters) would the earth have to be compressed in order for it to become a *black hole* - the escape velocity from its surface equal to the velocity $c = 3 \times 10^8 m/s$ of light?

(b) Repeat part (a) with the sun in place of the earth.

Section 2.4 - Numerical Approximation: Euler's Method

2.4.1 Apply Euler's method twice to approximate the solution to the initial value problem below on the interval $[0, \frac{1}{2}]$, first with step size $h = .25$, then with step size $h = 0.1$. Compare the three-decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value $y(\frac{1}{2})$ of the actual solution, also given below.

$$y' = -y,$$

$$y(0) = 2;$$

$$y(x) = 2e^{-x}.$$

More space, if necessary, for problem 2.4.1.

2.4.5 Apply Euler's method twice to approximate the solution to the initial value problem below on the interval $[0, \frac{1}{2}]$, first with step size $h = .25$, then with step size $h = 0.1$. Compare the three-decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value $y(\frac{1}{2})$ of the actual solution, also given below.

$$y' = y - x - 1,$$

$$y(0) = 1;$$

$$y(x) = 2 + x - e^x.$$

More room, if necessary, for problem 2.4.5.

2.4.9 Apply Euler's method twice to approximate the solution to the initial value problem below on the interval $[0, \frac{1}{2}]$, first with step size $h = .25$, then with step size $h = 0.1$. Compare the three-decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value $y(\frac{1}{2})$ of the actual solution, also given below.

$$y' = \frac{1}{4}(1 + y^2),$$

$$y(0) = 1;$$

$$y(x) = \tan\left(\frac{1}{4}(x + \pi)\right).$$

More room, if necessary, for problem 2.4.9.

2.4.26 Suppose the deer population $P(t)$ in a small forest initially numbers 25 and satisfies the logistic equation

$$\frac{dP}{dt} = 0.0225P - 0.0003P^2$$

(with t in months.) Use Euler's method with a programmable calculator or computer to approximate the solution for 10 years, first with step size $h = 1$ and then with $h = .5$, rounding off approximate P-values to integrals numbers of deer. What percentage of the limiting population of 75 deer has been attained after 5 years? After 10 years?

More room for problem 2.4.26.

2.4.30 Apply Euler's method with successively smaller step sizes on the interval $[0, 2]$ to verify empirically that the solution of the initial value problem

$$\frac{dy}{dx} = y^2 + x^2, \quad y(0) = 0$$

has vertical asymptote near $x = 2.003147$.

More room for problem 2.4.30.