Math 2280 - Assignment 14

Dylan Zwick

Fall 2013

Section 9.5 - 1, 3, 5, 7, 9

Section 9.6 - 1, 3, 5, 7, 14

Section 9.7 - 1, 2, 3, 4

Section 9.5 - Heat Conduction and Separation of Variables

9.5.1 - Solve the boundary value problem:

$$u_t = 3u_{xx}, \quad 0 < x < \pi, \quad t > 0;$$

$$u(0,t) = u(\pi,t) = 0, \quad u(x,0) = 4\sin(2x).$$

More room for Problem 9.5.1.

9.5.3 - Solve the boundary value problem:

$$u_t = 2u_{xx}, \quad 0 < x < 1, \quad t > 0;$$

$$u(0,t) = u(1,t) = 0, \quad u(x,0) = 5\sin(\pi x) - \frac{1}{5}\sin(3\pi x).$$

More room for Problem 9.5.3.

9.5.5 - Solve the boundary value problem:

$$u_t = 2u_{xx}, \quad 0 < x < 3, \quad t > 0;$$
$$u_x(0,t) = u_x(3,t) = 0, \quad u(x,0) = 4\cos\left(\frac{2}{3}\pi x\right) - 2\cos\left(\frac{4}{3}\pi x\right).$$

More room for Problem 9.5.5.

9.5.7 - Solve the boundary value problem:

$$3u_t = u_{xx}, \quad 0 < x < 2, \quad t > 0;$$

$$u_x(0,t) = u_x(2,t) = 0, \quad u(x,0) = \cos^2(2\pi x).$$

More room for Problem 9.5.7.

9.5.9 - Solve the boundary value problem:

$$10u_t = u_{xx}, \quad 0 < x < 5, \quad t > 0;$$

$$u(0,t) = u(5,t) = 0, \quad u(x,0) = 25.$$

More room for Problem 9.5.9.

Section 9.6 - Vibrating Strings and the One-Dimensional Wave Equation

9.6.1 - Solve the boundary value problem:

$$y_{tt} = 4y_{xx}, \quad 0 < x < \pi, \quad t > 0;$$

$$y(0,t) = y(\pi,t) = 0, \quad y(x,0) = \frac{1}{10}\sin(2x), y_t(x,0) = 0.$$

More room for Problem 9.6.1.

9.6.3 - Solve the boundary value problem:

$$4y_{tt} = y_{xx}, \quad 0 < x < \pi, \quad t > 0;$$

$$y(0,t) = y(\pi,t) = 0, \quad y(x,0) = y_t(x,0) = \frac{1}{10}\sin(x).$$

More room for Problem 9.6.3.

9.6.5 - Solve the boundary value problem:

$$y_{tt} = 25y_{xx}, \quad 0 < x < 3, \quad t > 0;$$

$$y(0,t) = y(3,t) = 0, \quad y(x,0) = \frac{1}{4}\sin(\pi x), y_t(x,0) = 10\sin(2\pi x).$$

More room for Problem 9.6.5.

9.6.7 - Solve the boundary value problem:

$$y_{tt} = 100 y_{xx}, \quad 0 < x < 1, \quad t > 0;$$

$$y(0,t) = y(1,t) = 0, \quad y(x,0) = 0, y_t(x,0) = x.$$

More room for Problem 9.6.7.

9.6.14 - Given the differentiable odd period $2L$ function $F(x)$, show that the function

$$y(x,t) = \frac{1}{2}\left[F\left(x+at\right) + F\left(x-at\right)\right]$$

satisfies the conditions

$$y(0,t) = y(L,t) = 0,$$
$$y(x,0) = F(x), y_t(x,0) = 0.$$

Section 9.7 - Steady-State Temperature and Laplace's Equation

9.7.1 - Solve the Dirichlet problem for the rectangle $0 < x < a, 0 < y < b$ consisting of Laplace's equation $u_{xx} + u_{yy} = 0$ and the boundary value conditions:

$$u(x,0) = u(x,b) = u(0,y) = 0, \quad u(a,y) = g(y).$$

More room for Problem 9.7.1.

9.7.2 - Solve the Dirichlet problem for the rectangle $0 < x < a, 0 < y < b$ consisting of Laplace's equation $u_{xx} + u_{yy} = 0$ and the boundary value conditions:

$$u(x, 0) = u(x, b) = u(a, y) = 0, \quad u(0, y) = g(y).$$

More room for Problem 9.7.2.

9.7.3 - Solve the Dirichlet problem for the rectangle $0 < x < a, 0 < y < b$ consisting of Laplace's equation $u_{xx} + u_{yy} = 0$ and the boundary value conditions:

$$u(x, 0) = u(0, y) = u(a, y) = 0, \quad u(x, b) = f(x).$$

More room for Problem 9.7.3.

9.7.4 - Consider the boundary value problem

$$u_{xx} + u_{yy} = 0; \; u_x(0, y) = u_x(a, y) = u(x, 0) = 0, \; u(x, b) = f(x)$$

corresponding to the rectangular plate $0 < x < a, 0 < y < b$ with the edges $x = 0$ and $x = a$ insulated. Derive the solution

$$u(x, y) = \frac{a_0 y}{2b} + \sum_{n=1}^{\infty} a_n \left(\cos\left(\frac{n\pi x}{a} \right) \right) \left(\frac{\sinh\left(\frac{n\pi y}{a}\right)}{\sinh\left(\frac{n\pi b}{a}\right)} \right), \text{ where}$$

$$a_n = \frac{2}{a} \int_0^a f(x) \cos\left(\frac{n\pi x}{a} \right) dx \quad (n = 0, 1, 2, \ldots).$$

(*Suggestion*: Show first that λ_0 is an eigenvalue with $X_0(x) \equiv 1$ and $Y_0(y) = y$.)

More room for Problem 9.7.4.