Math 2280 - Assignment 12

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Fall 2013

Section 8.4 - 1, 6, 8, 9, 14

Section 8.5 - 1, 5, 6, 13, 16

Section 8.4 - Method of Frobenius: The Exceptional Cases

8.4.1 - Either apply the method from Example 1 in the textbook to find two linearly independent Frobenius series solutions, or find one such solution and show (as in Example 2 from the textbook) that a second such solution does not exist for the differential equation:

$$xy'' + (3 - x)y' - y = 0.$$

More room for Problem 8.4.1, if you need it.

8.4.6 - Either apply the method from Example 1 in the textbook to find two linearly independent Frobenius series solutions, or find one such solution and show (as in Example 2 from the textbook) that a second such solution does not exist for the differential equation:

$$2xy'' - (6 + 2x)y' + y = 0.$$

More room for Problem 8.4.6, if you need it.

8.4.8 - Either apply the method from Example 1 in the textbook to find two linearly independent Frobenius series solutions, or find one such solution and show (as in Example 2 from the textbook) that a second such solution does not exist for the differential equation:

$$x(1-x)y'' - 3y' + 2y = 0.$$

More room for Problem 8.4.8, if you need it.

8.4.9 - For the differential equation

$$xy'' + y' - xy = 0,$$

first find the first four nonzero terms in a Frobenius series solution. Then use the reduction of order technique to find the logarithmic term and the first three nonzero terms in a second linearly independent solution.

More room for Problem 8.4.9, if you need it.

8.4.14 - For the differential equation

$$x^2y'' + x(1+x)y' - 4y = 0,$$

first find the first four nonzero terms in a Frobenius series solution. Then use the reduction of order technique to find the logarithmic term and the first three nonzero terms in a second linearly independent solution.

More room for Problem 8.4.14, if you need it.

Section 8.5 - Bessel's Equation

8.5.1 - Differentiate termwise the series for $J_0(x)$ to show directly that $J_0'(x) = -J_1(x)$ (another analogy with the cosine and sine functions).

8.5.5 - Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

8.5.6 - Derive the recursion formula:

$$[(m+r)^2 - p^2]c_m + c_{m-2} = 0$$

for Bessel's equation.

8.5.13 - Any integral of the form $\int x^m J_n(x)dx$ can be evaluated in terms of Bessel functions and the indefinite integral $\int J_0(x)dx$. The latter integral cannot be simplified further, but the function $\int_0^x J_0(t)dt$ is tabulated in Table 11.1 of Abramowitz and Stegun. Use the identities:

$$ds\tfrac{d}{dx}[x^p J_p(x)] = x^p J_{p-1}(x);$$

$$\frac{d}{dx}[x^{-p} J_p(x)] = -x^{-p} J_{p+1}(x);$$

to evaluate the integral

$$\int x^2 J_0(x)dx.$$

More room for Problem 8.5.13

8.5.16 - Same instructions as 8.5.13, only with the integral:

$$\int xJ_1(x)dx.$$

More room for Problem 8.5.16.