Math 2280 - Assignment 11

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Section 8.1 - 2, 8, 13, 21, 25

Section 8.2 - 1, 7, 14, 17, 32

Section 8.3 - 1, 8, 15, 18, 24

Section 8.1 - Introduction and Review of Power Series

8.1.2 - Find the power series solution to the differential equation

$$y' = 4y,$$

and determine the radius of convergence for the series. Also, identify the series solution in terms of familiar elementary functions.

More room for Problem 8.1.2, if you need it.

8.1.8 - Find the power series solution to the differential equation

$$2(x+1)y' = y,$$

and determine the radius of convergence for the series. Also, identify the series solution in terms of familiar elementary functions.

More room for Problem 8.1.8, if you need it.

8.1.13 - Find two linearly independent power series solutions to the differential equation

$$y'' + 9y = 0,$$

and determine the radius of convergence for each series. Also, identify the general solution in terms of familiar elementary functions.

More room for Problem 8.1.13, if you need it.

8.1.21 - For the initial value problem

$$y'' - 2y' + y = 0;$$

$$y(0) = 0, y'(0) = 1,$$

derive a recurrence relation giving c_n for $n \geq 2$ in terms of c_0 or c_1 (or both). Then apply the given initial conditions to find the values of c_0 and c_1. Next, determine c_n and, finally, identify the particular solution in terms of familiar elementary functions.

More room for Problem 8.1.21, if you need it.

8.1.25 - For the initial value problem

$$y'' = y' + y;$$
$$y(0) = 0, \quad y'(0) = 1,$$

derive the power series solution

$$y(x) = \sum_{n=1}^{\infty} \frac{F_n}{n!} x^n$$

where $\{F_n\}_{n=0}^{\infty}$ is the sequence $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ of *Fibonacci numbers* defined by $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-2} + F_{n-1}$ for $n > 1$.

More room for Problem 8.1.25, if you need it.

Even *more* room for Problem 8.1.25, if you need it.

Section 8.2 - Series Solutions Near Ordinary Points

8.2.1 - Find a general solution in powers of x to the differential equation

$$(x^2 - 1)y'' + 4xy' + 2y = 0.$$

State the recurrence relation and the guaranteed radius of convergence.

More room for Problem 8.2.1, if you need it.

8.2.7 - Find a general solution in powers of x to the differential equation

$$(x^2 + 3)y'' - 7xy' + 16y = 0.$$

State the recurrence relation and the guaranteed radius of convergence.

More room for Problem 8.2.7, if you need it.

8.2.14 - Find a general solution in powers of x to the differential equation

$$y'' + xy = 0.$$[1]

State the recurrence relation and the guaranteed radius of convergence.

[1] An *Airy equation*.

More room for Problem 8.2.14, if you need it.

8.2.17 - Use power series to solve the initial value problem

$$y'' + xy' - 2y = 0;$$

$$y(0) = 1, \quad y'(0) = 0.$$

More room for Problem 8.2.17, if you need it.

8.2.32 - Follow the steps outlined in this problem to establish *Rodrigues's formula*

$$P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$$

for the nth-degree Legendre polynomial.

(a) Show that $v = (x^2-1)^n$ satisfies the differential equation

$$(1-x^2)v' + 2nxv = 0.$$

Differentiate each side of this equation to obtain

$$(1-x^2)v'' + 2(n-1)xv' + 2nv = 0.$$

(b) Differentiate each side of the last equation n times in succession to obtain

$$(1-x^2)v^{(n+2)} - 2xv^{(n+1)} + n(n+1)v^{(n)} = 0.$$

Thus $u = v^{(n)} = D^n(x^2-1)^n$ satisfies Legendre's equation of order n.

(c) Show that the coefficient of x^n in u is $(2n)!/n!$; then state why this proves Rodrigues' formula. (Note that the coefficient of x^n in $P_n(x)$ is $(2n)!/[2^n(n!)^2]$.)

More room for Problem 8.2.32, if you need it. You probably will.

Even *more* room for Problem 8.2.32, if you need it.

Section 8.3 - Regular Singular Points

8.3.1 - Determine whether $x = 0$ is an ordinary point, a regular singular point, or an irregular singular point for the differential equation

$$xy'' + (x - x^3)y' + (\sin x)y = 0.$$

If it is a regular singular point, find the exponents of the differential equation (the solutions to the indicial equation) at $x = 0$.

8.3.8 - Determine whether $x = 0$ is an ordinary point, a regular singular point, or an irregular singular point for the differential equation

$$(6x^2 + 2x^3)y'' + 21xy' + 9(x^2 - 1)y = 0.$$

If it is a regular singular point, find the exponents of the differential equation (the solutions to the indicial equation) at $x = 0$.

8.3.15 - If $x = a \neq 0$ is a singular point of a second-order linear differential equation, then the substitution $t = x - a$ transforms it into a differential equation having $t = 0$ as a singular point. We then attribute to the original equation at $x = a$ the behavior of the new equation at $t = 0$. Classify (as regular or irregular) the singular points of the differential equation

$$(x-2)^2 y'' - (x^2-4)y' + (x+2)y = 0.$$

8.3.18 - Find two linearly independent Frobenius series solutions (for $x > 0$) to the differential equation

$$2xy'' + 3y' - y = 0.$$

More room for Problem 8.3.18, if you need it.

8.3.24 - Find two linearly independent Frobenius series solutions (for $x > 0$) to the differential equation

$$3x^2y'' + 2xy' + x^2y = 0.$$

More room for Problem 8.3.24, if you need it.