Chapter 1

Quizzes

1.1 Number and Operation

Before we get started on algebra, let's check our understanding of the numbers we will be studying.

1. Mark the following numbers as *integers* (**Z**), *rational numbers* (**Q**), or *real numbers* (**R**). Many will belong to more than one group.

   (a) 3

   (b) $\dfrac{4}{3}$

   (c) $\dfrac{6}{3}$

   (d) 1.3

   (e) $1.\overline{3}$
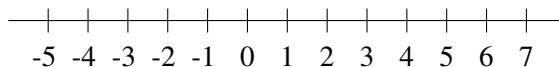
   (f) $\sqrt{2}$

   (g) 4.0

   (h) π

   (i) -6

   (j) $-\dfrac{1}{2}$

2. Plot the following values on the provided number line:

$$A = 5.0, B = -1/2, C = \sqrt{2}, D = -4.3, E = \pi$$

3. Evaluate the following expressions. Write any fractions in lowest terms.

   (a) $-(-4)$

   (b) $\dfrac{9-5}{6}$

   (c) $-|5|$

   (d) $-|-5|$

   (e) $|7-5|$

4. Perform the following computations, leaving answers in lowest terms, where applicable.

   (a) $\dfrac{2}{3} + \dfrac{4}{5}$

   (b) $125 - (-327)$

   (c) $4\dfrac{3}{4} - 3\dfrac{7}{8}$

   (d) $4\dfrac{3}{4} \cdot 3\dfrac{7}{8}$

   (e) $1.5 \cdot 2.25$

   (f) $2.25 \div 1.5$

   (g) $-81 \div 3$

   (h) $(-91) \div (-13)$

5. Perform the following computations involving exponents:

   (a) 2^4

   (b) 2^{-4}

   (c) 9^2

   (d) $9^{\frac{1}{2}}$

   (e) $9^{-\frac{1}{2}}$

   (f) -4^2

   (g) $(-4)^2$

6. Perform the following computations. Be sure to observe the correct order of operations.

   (a) $6-3+2$

   (b) $6 \div 3 \cdot 2$

   (c) $(14-5)^2$

   (d) $\dfrac{4^3 - 2 \cdot 5}{6 - 4 \div 2}$

7. Find the distance between the given numbers along the number line.

   (a) $5, 2$

   (b) $-5, 2$

   (c) $-5, -2$

   (d) $5, -2$

1.2 Expressions

1. Simplify the following expressions:

   (a) $2x + 3y - (x + y)$

   (b) $-3x + \dfrac{y}{2} - (y - 2x)$

   (c) $\dfrac{2x}{4}$

2. Write an expression for each of the following:

   (a) The number of eggs I have total if I have d dozen and e more eggs.

   (b) The amount of money I have left if I started with d dimes and spent c cents.

1.3 Polynomial expressions

1.3.1 Multiplying

Multiply out and write in standard form:

1. $(x-2)(5x-2)$

2. $(x-1)^2(x+2)$

3. $(x-2)^3$

4. $3(x-2)(x+3)$

5. $(5x+3)(x^2 - 3x - 4)$

6. $(9 - 6i)^2$ (here $i^2 = 1$)

1.3.2 Factoring

Factor as completely as possible:

1. $x^2 + 7xy - 144y^2$

2. $10x^2 + 6xy - 25xy - 15y^2$

3. $3x^3y^2 - 75xy^2$

4. $36x^3 + 12x^2 - 48x$

5. $x^3 + 3x^2 - x - 3$

6. $x^4 - y^4$

You'll get lots more factoring practice in later sections.

1.4 Rational expressions

Simplify the following rational expressions as completely as possible:

1. $\dfrac{2x^2 + 5x - 3}{x^2 + 2x - 3}$

2. $\dfrac{-4x^3y^2}{2x^2y^3} \cdot \dfrac{-4xy^3}{10xy^2}$

3. $\dfrac{3}{x-2} + \dfrac{3x}{2x+3}$

4. $\dfrac{4x - 48}{x^2 - 144}$

5. $\dfrac{x^2 - 9}{2x + 2} \cdot \dfrac{x^2 + 2x + 1}{(x-3)(x+1)}$

6. $\dfrac{x^2 + x - 2}{3x^2 - 9x} \div \dfrac{x^2 + 4x - 5}{6x - 18} \cdot \dfrac{x^2 + 2x - 15}{x^2 - x - 6}$

1.4.1 Complex fractions

It's not considered good mathematical writing to have fractions living inside of other fractions. Such expressions are called *complex fractions*. Have no fear: a fraction bar signifies division, so one fraction over another can be rewritten as a division of fractions problem.
Write these as simple fractions:

1. $\dfrac{\dfrac{x+5}{3x^2}}{\dfrac{x^2-25}{6x^3}}$

2. $\dfrac{4 + \dfrac{1}{2}}{\dfrac{1}{3} + \dfrac{1}{6}}$

3. $\dfrac{\dfrac{5}{x-5} + \dfrac{3}{x+3}}{\dfrac{5}{x+3} + \dfrac{3}{x-5}}$

1.5 Radical expressions

Simplify the following expressions involving radicals completely.

1. $3\sqrt{75} + 4\sqrt{12}$

2. $\left(\sqrt{14x^3y}\right)\left(\sqrt{7x^3y^3}\right)$

3. $\sqrt{\dfrac{375x^5}{5x}}$

4. $\left(4\sqrt{5} - 2\right)\left(2\sqrt{5} + 4\right)$

5. $\sqrt[3]{xy^5}\sqrt[3]{x^7y^{17}}$

6. $\dfrac{\sqrt[5]{320x^{13}y^{18}}}{\sqrt[5]{10x^3y^3}}$

7. $\sqrt{16x^2 + 4}$

Simplify these expressions involving complex numbers and write them in standard form.

1. i^{37}

3. $\dfrac{2+3i}{4-i}$

2. $(2+3i)(4-i)$

4. $\sqrt{-32}$

1.6 Linear equations and inequalities

1.6.1 Equations

Solve for x:

1. $3x+4=6-x$

5. $\dfrac{3+x}{5}=\dfrac{7}{10}$

2. $-3x+4=6-x$

6. $\dfrac{3-x}{x}=\dfrac{3}{5}$

3. $\dfrac{3}{x}=\dfrac{9}{14}$

7. $\dfrac{x-2}{4}=\dfrac{x+1}{12}$

4. $\dfrac{x}{7}=\dfrac{3}{5}$

1.6.2 Harder Equations

Solve for x:

1. $\dfrac{x}{3}-2=\dfrac{3}{10}$

4. $\dfrac{3x+3}{3}=\dfrac{2x+2}{2}$

2. $\dfrac{2x-3}{5}+x=\dfrac{4-x}{4}+3$

5. $\dfrac{3x+3}{3}=\dfrac{2x+3}{2}$

3. $\dfrac{2}{x-1}=\dfrac{8}{3x+6}$

6. $\dfrac{1}{x+3}-2=5$

1.6.3 Inequalities

Solve for x, writing your answer in interval notation:

1. $3x+4<6-x$

4. $\dfrac{3+x}{5}\le\dfrac{7}{10}$

2. $-3x+4\ge6-x$

5. $\dfrac{x-2}{4}\ge\dfrac{x+1}{12}$

3. $\dfrac{x}{7}<\dfrac{3}{5}$

6. $-3<2x+5\le7$

1.7 Systems of linear equations

Find the values of x and y that satisfy the following equations simultaneously.

1.
$$\begin{cases} 2x + 3y = 7 \\ y = 4 \end{cases}$$

3.
$$\begin{cases} 2x - 3y = 5 \\ 6x - 9y = 4 \end{cases}$$

2.
$$\begin{cases} 0.4x + 0.6y = 3.2 \\ 1.4x - 0.3y = 1.6 \end{cases}$$

4.
$$\begin{cases} 2x - 3y = 5 \\ -6x + 9y = -15 \end{cases}$$

5.
$$\begin{cases} 2x & - & 3y & + & z & = & 5 \\ & & 2y & + & z & = & 4 \\ & & & & z & = & -2 \end{cases}$$

6.
$$\begin{cases} 2x & - & 3y & + & z & = & 1 \\ x & + & 2y & + & z & = & -1 \\ 3x & - & y & + & 3z & = & 4 \end{cases}$$

1.8 Absolute Value Equations and Inequalities

1.8.1 Equations

Solve the following for x, noting that some equations may have multiple (or no) solutions.

1. $|3x-2|=6$

2. $\left|\dfrac{3-x}{3}\right|-4=-2$

3. $|x+5|+7=5$

4. $2-\left|\dfrac{x}{5}-1\right|=1$

5. $|2x+3|=|3x-2|$

1.8.2 Inequalities

Solve the following for x, writing your answers in interval notation. Then graph your solutions.

1. $|3x-4|<6$

2. $|4-x|\geq 3$

3. $|2-3x|\leq 6$

4. $|-x-5|+5\leq 3$

5. $|2x+5|>-3$

6. $2<\left|\dfrac{x+2}{3}-1\right|<4$

1.9 Functions and graphs

1.9.1 Functions

Do the following equations, tables, or graphs represent y as a function of x?

1. $y=2x-3$

2. $3x-2y^2=-3$

3. $x^2+y^5=-1$

4. $x=\sqrt{y}+1$

6.

x	y
-2	4
-1	4
0	4
1	4
2	4

5.

x	y
-2	4
-1	1
0	0
1	1
2	4

7.

x	y
-1	4
-1	1
0	0
1	1
1	4
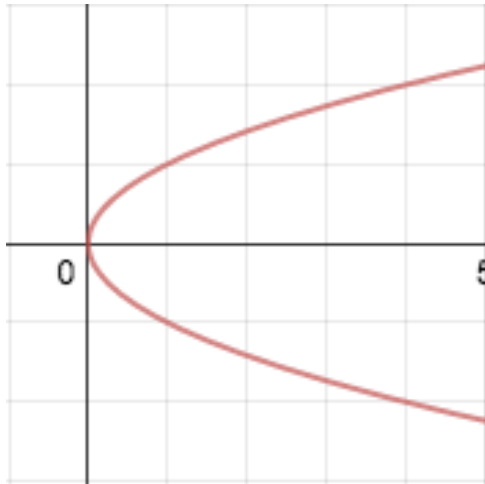
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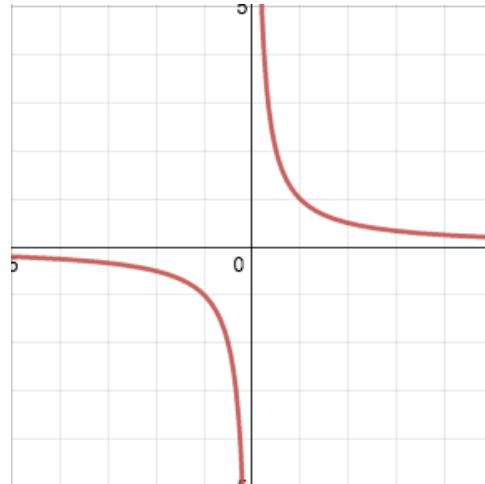
9.

For the next problems,

$$f(x) = \frac{1}{1-x}, g(x) = x^2 + 1$$

1. Find the domain of $f(x)$.

2. Find the domain of $g(x)$.

3. Find and simplify $f(x)/g(x)$, and give its domain.

4. Find and simplify $g(x)/g(x)$, and give its domain.

5. Find and simplify $(f \circ g)(x)$ and give its domain.

6. Find and simplify $(g \circ f)(x)$ and give its domain.

7. Evaluate $f(-2)$.

8. Evaluate $g(-2)$.

9. Evaluate $f(1/x)$ and write the result as a simple fraction.

10. Find $f^{-1}(x)$

11. Why does g not have an inverse?

12. Find the inverse of the function

$$h(x) = \frac{2x-3}{x+5}$$

1.9.2 Translation and reflection of graphs

Sketch the graphs of the following:

1. $y = -(x+2)^2 - 3$

2. $y = (x-3)^2 + 5$

3. $y = |x-1| + 4$

4. $y = (x-2)^3 - 3$

5. $y = -(x+2)^3$

6. $y = \dfrac{-1}{x+3}$

1.10 Graphs of linear equations and inequalities

1.10.1 Drawing graphs

Sketch the graphs of the following linear equations or inequalities.

1. $y = 2x + 3$

2. $y = -\dfrac{1}{2}x - 2$

3. $y = 2 - 3x$

4. $2x + 3y = 6$

5. $x = 4$

6. $0.2x + 0.6y = 1.2$

7. $y < -\dfrac{1}{2}x + 4$

8. $2x - 4y \geq 3$

9. $x > 2y - 5$

1.10.2 Finding equations of lines

Find the equations of lines (in slope-intercept form)

1. slope=-2, y-intercept (0,-1)

2. through the points (-1,3) and (2,-2)

3. through the points (1,1) and (1,-5)

4. with y-intercept $(0, 3)$ and parallel to $y = 2x - 4$

5. through (-1,1) and perpendicular to $2x + y = 5$

6. through (2,-3) and perpendicular to $y = 4$

1.11 Polynomial equations

Solve for x:

1. $x^2 - 2x + 1 = 0$

2. $x^2 - 8x + 7 = 0$

3. $x^2 - 5x = 6$

4. $x^2 - 2x = 0$

5. $x^2 - \dfrac{1}{4} = 0$

6. $x^2 + 16 = 0$

7. $3x^2 + 6x = 24$

8. $2x^2 - 3x - 2 = 0$

9. $6x^2 = 7x + 3$

10. $12x^2 - 13x + 3 = 0$

11. $x^4 - 2x^2 + 1 = 0$

12. $x - 8\sqrt{x} + 7 = 0$

13. $x^3 - 5x^2 = 6x$

14. $x^4 - 4x^2 = 0$

15. $(x - 2)^2 = 7$

16. $x^2 - 2x - 5 = 0$

17. $x^2 + 7x + 3 = 0$

18. $x^2 + 2x + 3 = 0$

19. $x^2 + 14x + 49 = 0$

20. $x^2 - 14x + 49 = 0$

1.12 Graphs of quadratic functions

1. Write $y = 2(x - 1)^2 + 4$ in the form $y = ax^2 + bx + c$.

2. Write $y = 2x^2 + 4x + 5$ in the form $y = a(x - h)^2 + k$.

3. Find the coordinates of the vertex of the parabola $y = 2(x - 1)^2 + 4$

4. Find the coordinates of the vertex of the parabola $y = 2x^2 + 4x + 5$.

5. Find the equation of a parabola with vertex $(-1, 3)$ with y-intercept $(0, -2)$.

6. Find the equation of a parabola with roots -2 and 3.

7. Graph $y = 2(x - 1)^2 + 4$.

8. Graph $y = 2x^2 + 4x + 5$.

1.13 Equations involving radicals

Solve for the variable:

1. $3\sqrt{5-2x} = 9$

2. $\sqrt{2x+1}+1 = 4$

3. $y-2 = \sqrt{y+4}$

These next two are pretty hard. They're probably a bit above 1010 level. Try them!

1. $\sqrt{5t} = 1+\sqrt{5(t-1)}$

2. $\sqrt{1+6x} = 2-\sqrt{6x}$

1.14 Exponential and Logarithmic functions

Evaluate:

1. $8^{2/3}$

2. $8^{-2/3}$

3. $(-8)^{2/3}$

4. $(-8)^{-2/3}$

5. $-8^{2/3}$

6. $\left(\dfrac{4}{9}\right)^{3/2}$

7. $\log_2(1/2)$

8. $\log_3 27$

9. $\log_9 3$

Solve:

1. $\log x = -3$

2. $\log_2 x = -\dfrac{1}{2}$

3. $e^{2x} - 2 = 0$

4. $8 - 3 \cdot 2^{0.5x} = -40$

5. $\log_3 9x = 3$

6. $e^{2x} - 3e^x = 28$

7. $\log(x+1) - \log(x-1) = \log 3$

8. $\ln(x^2-4) - \ln(x+2) = ln(3-x)$

Combine the logarithms into a single logarithm:

1. $\dfrac{1}{2}\log_2 x + log_2 y$

2. $3\log x - 2\log y - \log z$

Expand the single logarithm as fully as possible:

1. $\log\left(\dfrac{x\sqrt{y}}{z}\right)$

2. $\log_4 4x^2 y^3$