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Maple Project 5 Example Writeup

```
interface(displayprecision = 2) :
with(LinearAlgebra) :
L := Matrix([[0, 1, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0], [0, 1, 1, 0, 1, 0, 0], [0, 0, 0,
    0, 0, 0, 1], [1, 1, 0, 0, 1, 0, 0], [1, 0, 1, 1, 1, 1, 0]]);
```

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad (1)$$

```
h[0] := L.Vector(7, 1);
```

$$\begin{bmatrix} 2 \\ 1 \\ 1 \\ 3 \\ 1 \\ 3 \\ 5 \end{bmatrix} \quad (2)$$

```
Normalize(h[0], inplace);
```

$$\begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \\ \frac{1}{5} \\ \frac{3}{5} \\ \frac{1}{5} \\ \frac{3}{5} \\ 1 \end{bmatrix} \quad (3)$$

```
h[1] := L.L^+.Vector(7, 1);
```

$$\begin{bmatrix} 6 \\ 3 \\ 4 \\ 9 \\ 2 \\ 10 \\ 10 \end{bmatrix} \tag{4}$$

$Normalize(h[1], inplace);$

$$\begin{bmatrix} \frac{3}{5} \\ \frac{3}{10} \\ \frac{2}{5} \\ \frac{9}{10} \\ \frac{1}{5} \\ 1 \\ 1 \end{bmatrix} \tag{5}$$

$H := \langle h[0] | h[1] \rangle;$

$$\begin{bmatrix} \frac{2}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{3}{10} \\ \frac{1}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{9}{10} \\ \frac{1}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \\ 1 & 1 \end{bmatrix} \tag{6}$$

$a[0] := L^{+}.Vector(7, 1);$

$$\begin{bmatrix} 3 \\ 4 \\ 2 \\ 1 \\ 3 \\ 1 \\ 2 \end{bmatrix} \tag{7}$$

Normalize(*a*[0], *inplace*);

$$\begin{bmatrix} \frac{3}{4} \\ 1 \\ \frac{1}{2} \\ \frac{1}{4} \\ \frac{3}{4} \\ \frac{1}{4} \\ \frac{1}{2} \end{bmatrix} \tag{8}$$

$a[1] := L^{+}.L.Vector(7, 1);$

$$\begin{bmatrix} 9 \\ 9 \\ 8 \\ 5 \\ 11 \\ 5 \\ 3 \end{bmatrix} \tag{9}$$

Normalize(*a*[1], *inplace*);

$$\begin{bmatrix} \frac{9}{11} \\ \frac{9}{11} \\ \frac{8}{11} \\ \frac{5}{11} \\ 1 \\ \frac{5}{11} \\ \frac{3}{11} \end{bmatrix} \quad (10)$$

$A := \langle a[0] | a[1] \rangle;$

$$\begin{bmatrix} \frac{3}{4} & \frac{9}{11} \\ 1 & \frac{9}{11} \\ \frac{1}{2} & \frac{8}{11} \\ \frac{1}{4} & \frac{5}{11} \\ \frac{3}{4} & 1 \\ \frac{1}{4} & \frac{5}{11} \\ \frac{1}{2} & \frac{3}{11} \end{bmatrix} \quad (11)$$

for i **from** 2 **to** 6 **do** $h[i] := L.L^{+}.h[i-2] : Normalize(h[i], inplace) : H := \langle H | h[i] \rangle : a[i] := L^{+}.L.a[i-2] : Normalize(a[i], inplace) : A := \langle A | a[i] \rangle$ **end do**:
$map(evalf, H);$

$$\begin{bmatrix} 0.40 & 0.60 & 0.32 & 0.41 & 0.30 & 0.34 & 0.29 \\ 0.20 & 0.30 & 0.24 & 0.25 & 0.24 & 0.25 & 0.24 \\ 0.20 & 0.40 & 0.24 & 0.32 & 0.25 & 0.28 & 0.25 \\ 0.60 & 0.90 & 0.74 & 0.85 & 0.76 & 0.81 & 0.77 \\ 0.20 & 0.20 & 0.08 & 0.09 & 0.05 & 0.06 & 0.04 \\ 0.60 & 1.00 & 0.76 & 0.89 & 0.80 & 0.84 & 0.80 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix} \quad (12)$$

map(evalf, A);

$$\begin{bmatrix} 0.75 & 0.82 & 0.79 & 0.80 & 0.78 & 0.80 & 0.79 \\ 1.00 & 0.82 & 1.00 & 0.82 & 0.90 & 0.82 & 0.86 \\ 0.50 & 0.73 & 0.66 & 0.69 & 0.67 & 0.69 & 0.68 \\ 0.25 & 0.45 & 0.34 & 0.40 & 0.37 & 0.39 & 0.38 \\ 0.75 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 0.25 & 0.45 & 0.34 & 0.40 & 0.37 & 0.39 & 0.38 \\ 0.50 & 0.27 & 0.28 & 0.16 & 0.18 & 0.13 & 0.15 \end{bmatrix} \quad \textbf{(13)}$$

 # The final columns in the above matrices are reasonable estimates for h and a, respectively.
 # From these estimates it looks like node 7 is the best hub, and it is the site with the most outgoing links.
 # However, it looks like node 5 is the best authority, and it is NOT the site with the most incoming links. The site with the most incoming links is site 2.

 # We can check our estimate for h:
$h := \langle .29, .24, .25, .77, .04, .80, 1.00 \rangle;$

$$\begin{bmatrix} 0.29 \\ 0.24 \\ 0.25 \\ 0.77 \\ 0.04 \\ 0.80 \\ 1.00 \end{bmatrix} \quad \textbf{(14)}$$

$Normalize(L.L^{+}.h, inplace);$

$$\begin{bmatrix} 0.29 \\ 0.24 \\ 0.25 \\ 0.77 \\ 0.04 \\ 0.80 \\ 1.00 \end{bmatrix} \quad \textbf{(15)}$$

 # We see that h is a very good approximation of an eigenvector of LL^{+}. *We now estimate the eigenvalue.*

$\frac{norm(L.L^{+}.h, 2)}{norm(h, 2)};$

$$8.39 \quad \textbf{(16)}$$

 # So, it looks like the leading eigenvalue is about 8.39.
 # We can do the same for a, estimating a as the last column of A.
$a := \langle .79, .86, .68, .38, 1, .38, .15 \rangle;$

$$\begin{bmatrix} 0.79 \\ 0.86 \\ 0.68 \\ 0.38 \\ 1 \\ 0.38 \\ 0.15 \end{bmatrix} \quad (17)$$

Normalize$(L^{+}.L.a, inplace)$;

$$\begin{bmatrix} 0.79 \\ 0.84 \\ 0.69 \\ 0.38 \\ 1.00 \\ 0.38 \\ 0.14 \end{bmatrix} \quad (18)$$

 # Again, we see a is a good approximation of an eigenvector of $L^{+}.L$. *We can estimate the eigenvalue*

$\dfrac{norm(L^{+}.L.a, 2)}{norm(a, 2)}$;

$$8.39 \quad (19)$$

#So, it appears the leading eigenvalue is approximately 8.39.