Math 2270 - Lecture 1: Vectors

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1 Vectors

Linear algebra begins with the study of *vectors*. A vector is an ordered set of numbers, usually written in a rectangular array like this:

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

The number of elements in the array indicates the *dimension* of the space in which the vector lives.

1.1 Visualizing Vectors

Vectors can be visualized as representing arrows in an appropriate space. For example, the vector above can be visualized as representing the vector pointing from the origin to the point $(3, 2)$ in the xy-plane.
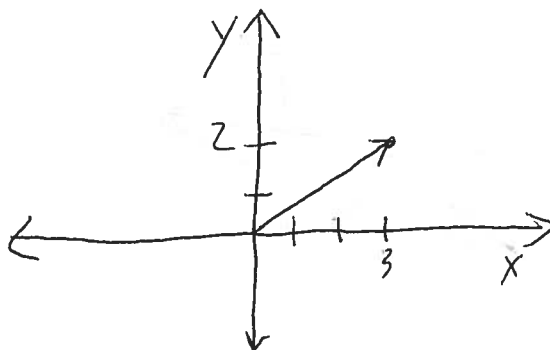
Now, vectors don't always have to begin at the origin. For example,

we could have a vector that points from the point $(1, 2)$ to the point $(4, 4)$. This vector is pictured below.
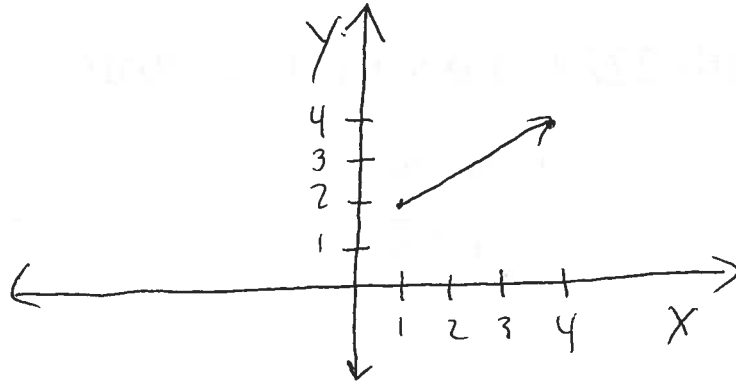

In both these examples, the tip of the vector is three units to the right and two units up from the tail of the vector. Now, we don't want vectors to be tied down to a particular starting point, and so we'd say these two vectors are in fact the same vector.
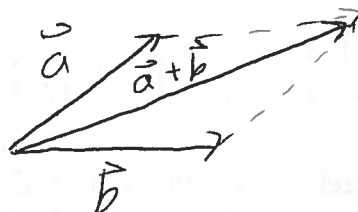
1.2   Adding Vectors

We add vectors componentwise. For example

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}.$$

We can view this addition visually using either the *tail to tip* method:


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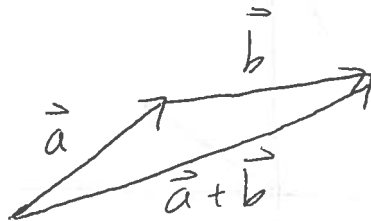
Or the *parallelogram method*:

Both these methods work and give you the same answer, as they are both really the same method.

1.3 Multiplying Vectors

We don't in general multiply vectors componentwise the way we add vectors. However, we can multiply a vector by a *scalar* (which, in this class, will almost always be a real number) to get another vector. We do this componentwise. So, for example:

$$2\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}.$$

As the name "scalar" might suggest, what this does visually is it scales the vector, giving you a vector pointing in the same direction (if the scalar is positive), or the opposite direction (if the scalar is negative) with length scaled by the magnitude of the scalar.

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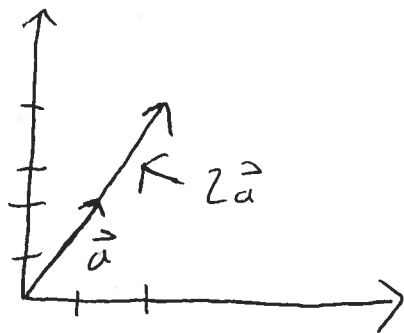
1.4 Linear Combinations

A linear combination of vectors is a vector sum of scalar multiples. So, for example:

$$2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}.$$

Frequently in linear algebra we'll be interested in not just a particular linear combination of two vectors, but in the set of *all* linear combinations we can make with a given set of vectors, and what these sets look like. For example, *any* point in the xy-plane can be written as a linear combination of the above two vectors. But, (much) more on this later.

1.5 Higher Dimensions

All of our examples so far have been in two dimensions. However, linear algebra extends naturally into three or even more dimensions. (Extending the visual representations into higher dimensions becomes difficult after three!) For example, we can easily add two vectors in three dimensions:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}.$$