

# Math 2270 - Review Subjects for the Final

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This is a list of the general subjects you should know for the final exam. It covers the major themes we've studied in the class. Not all these subjects will be on the final, but they're all fair game.

- **Matrix Arithmetic**

- Adding matrices.
- Multiplying matrices.
- When these operations can be performed.

- **Vector Spaces and Subspaces**

- Requirements of a subspace.
  - \* Closed under addition.
  - \* Closed under scalar multiplication.
- Calculating nullspaces.
- Calculating the complete solution to  $Ax = \mathbf{b}$ .
- Calculating the rank of a matrix.
- The four fundamental subspaces.
  - \* Their relative dimensions.
  - \* Which ones are orthogonal.

- **Projections**

- Projecting a vector onto another. Projecting  $\mathbf{a}$  onto  $\mathbf{b}$  is  $\left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}}\right) \mathbf{b}$ .
- Projecting a vector onto the subspace spanned by the columns of a matrix  $A$ . Solve  $A^T A \hat{\mathbf{x}} = A^T \mathbf{b}$ . The projection is  $\mathbf{p} = A \hat{\mathbf{x}} = A(A^T A)^{-1} A^T \mathbf{b}$ . The projection matrix is  $P = A(A^T A)^{-1} A^T$ .
- Calculate the least-squares linear approximation.
- The Gram-Schmidt method.

- **Determinants**

- How to calculate determinants.
  - \* Big formula.
  - \* Cofactor expansion.
  - \* Product of pivots.
  - \* Product of eigenvalues.
- If  $\det(A) \neq 0$  then  $A$  is invertible.

- **Eigenvalues**

- Finding the characteristic polynomial of a matrix.
- Eigenvalues are the roots of the characteristic polynomial.
- Diagonalizing a matrix.
- Ways to know a matrix is diagonalizable.
  - \* If all the eigenvalues are distinct.
  - \* If the matrix is symmetric.
- Symmetric matrices
  - \* All real eigenvalues.
  - \* Orthogonal eigenvectors.
  - \* Always diagonalizable.
  - \*  $A = Q \Lambda Q^T$ .
- Positive definite matrices.
  - \* Definition - Symmetric, all positive eigenvalues.
  - \* All positive pivots.

- \* Upper-left determinants all positive.
- \*  $A = R^T R$  and  $R$  has independent columns.
- Similar matrices
  - \*  $B = M^{-1} A M$ , where  $M$  is invertible.
  - \* Jordan canonical form.
- Singular value decomposition.
  - \* How to calculate.
  - \* How to calculate a pseudoinverse.

- **Linear Transformations**

- Definition.
- How to check if a function is a linear transformation.  $T(\mathbf{x} + \mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y})$ , and  $T(c\mathbf{x}) = cT(\mathbf{x})$ .

- **Directed Graphs**

- Calculating an incidence matrix.
- Eliminating produces a spanning tree.

- **Markov Matrices**

- Definition (All columns sum to 1), all entries non-negative. A positive Markov matrix has all entries positive.
- $\lambda = 1$  is the largest eigenvalue, and a positive eigenvector can be found for it.
- If it's a positive Markov matrix, all eigenvalues have magnitude  $\leq 1$ .
- The positive eigenvector for the eigenvalue  $\lambda = 1$  is the steady state solution.