

Math 2270 - Practice Final

University of Utah

Fall 2012

Name: _____

This is a 2 hour exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. *Elimination and LU Factorization*

- (a) (20 points) Use elimination to calculate the row-echelon form of the matrix

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & 7 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

- (b) (15 points) Calculate the LU factorization of the matrix from part (a)

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & 7 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

- (c) (5 points) Is the matrix A positive-definite? How do you know?

2. (a) (15 points) What is the rank of the matrix?

$$\begin{pmatrix} 2 & 3 & 1 & 4 \\ 2 & 4 & 3 & 5 \\ 2 & 5 & 5 & 6 \\ 0 & 3 & 6 & 3 \end{pmatrix}$$

(b) (20 points) What is the nullspace of the matrix? (Express it as the span of a set of linearly independent vectors.)

3. (15 points) Calculate the determinant of the matrix

$$\begin{pmatrix} 2 & 3 & 5 \\ 1 & 2 & 1 \\ 3 & 1 & 4 \end{pmatrix}$$

Is the matrix invertible? Why or why not?

4. (15 points) Use the Gram-Schmidt process to find an orthonormal basis for the vector space spanned by the vectors

$$\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}.$$

5. (a) (20 points) Calculate the eigenvalues of the matrix

$$\begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

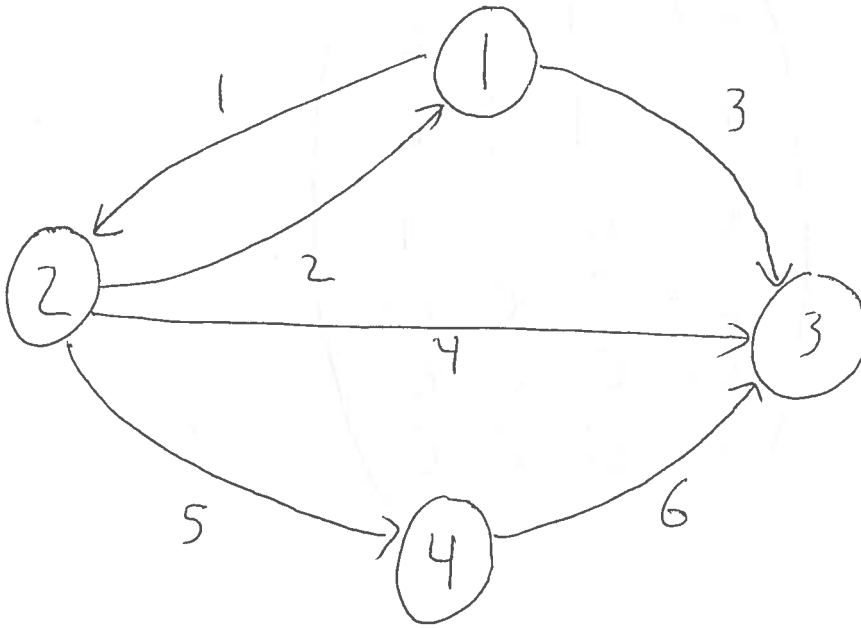
- (b) (20 points) Find the diagonalization, $A = S\Lambda S^{-1}$, of the matrix from part (a).

$$\begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

6. (15 points) What are the possible Jordan canonical forms for a 5×5 matrix with eigenvalues $\lambda = 1, 1, 1, 2, 2$.

7. (10 points) Is the transformation $T(v_1, v_2) = v_1v_2 + v_1 + v_2$ a linear transformation? Why or why not?

8. (a) What is the incidence matrix for the graph:



(b) (10 points) Perform elimination on the incidence matrix from part (a).

(c) (10 points) What is the spanning tree for the graph from part (a) determined by the reduction of the incidence matrix calculated in part (b)?