

Math 2270 - Final

University of Utah

Fall 2012

Name: _____

This is a 2 hour exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

Problems 3 and 5 are fairly involved. You're only expected to do one of them, and the exam will be graded out of 200 points. You can decide which of the two you would like to do. Please check the option you'd like for each of the two problems. If you finish the exam early and would like to do the other problem you may, and it will be graded for extra credit, but it will only be worth 20 points instead of 50.

1. *Elimination and LU Factorization* (35 points total)

- (a) (15 points) Use elimination to calculate the row echelon form of the matrix

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{pmatrix}$$

- (b) (15 points) Calculate the LU factorization of the matrix A from part (a) of this problem, given below

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{pmatrix}$$

- (c) (5 points) Is the matrix A invertible? How do you know?

2. *Subspaces* (20 points total)

(a) (10 points) Is the set of all vectors (a_1, a_2, a_3) with $a_3 = 1$ a subspace of \mathbb{R}^3 ?

(b) (10 points) Is the set of all vectors (a_1, a_2, a_3) with $a_3 = 0$ a subspace of \mathbb{R}^3 ?

3. *Elimination, Rank, and Complete Solutions*

Check the option you'd like:

_____ Full Credit (50 points total)

_____ Extra Credit (20 points total)

(a) (10 points or 4 points extra credit) Use elimination to calculate the row echelon form of the matrix

$$B = \begin{pmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{pmatrix}$$

(b) (5 points or 2 points extra credit) What is the rank of the matrix B from part (a) of this problem?

(c) (10 points or 4 points extra credit) What are the dimensions of the following subspaces for the matrix B :

- Row space $\mathbf{C}(B^T)$ - _____
- Nullspace $\mathbf{N}(B)$ - _____
- Column space $\mathbf{C}(B)$ - _____
- Left nullspace $\mathbf{N}(B^T)$ - _____

- (d) (10 points or 4 points extra credit) What is a basis for the nullspace, $\mathbf{N}(B)$, of the matrix B from part (a) of this problem?

(e) (15 points or 6 points extra credit) Find the complete solution to the system of equations

$$\begin{aligned}2x_1 + 4x_2 + 6x_3 + 4x_4 &= 4 \\2x_1 + 5x_2 + 7x_3 + 6x_4 &= 5 \\2x_1 + 3x_2 + 5x_3 + 2x_4 &= 3\end{aligned}$$

4. *Projections* (15 points)

Calculate the projection of the vector **a** onto the vector **b**:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ -2 \\ 4 \end{pmatrix}$$

5. *Eigenvalues and Diagonalization*

Check the option you'd like:

_____ Full Credit (50 points total)

_____ Extra Credit (20 points total)

(a) Calculate the eigenvalues of the matrix (15 points or 6 points extra credit)

$$C = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$$

- (b) Calculate normalized eigenvectors for the matrix C from part (a).
(15 points or 6 points extra credit)

(c) Diagonalize, $C = S\Lambda S^{-1}$, the matrix C from part (a). (15 points or 6 points extra credit)

(d) Is the matrix C positive-definite? Why or why not? (5 points or 2 points extra credit)

6. *Singular Value Decomposition and the Pseudoinverse* (35 points total)

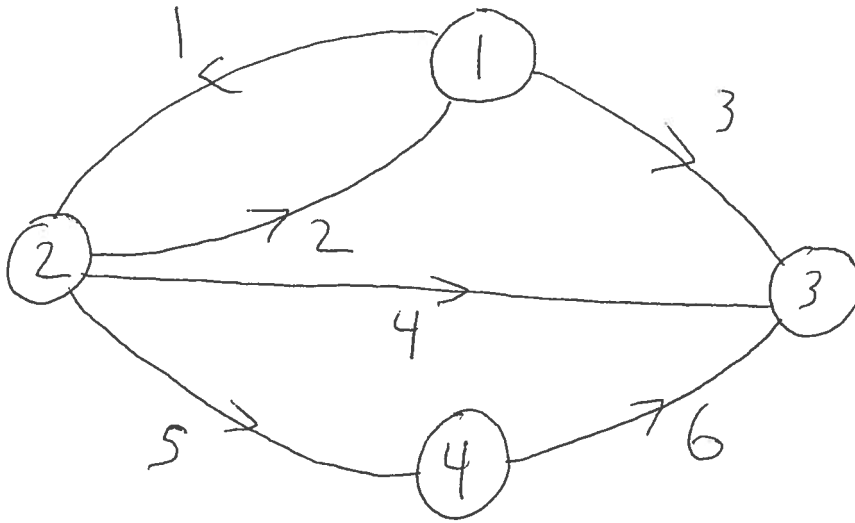
(a) Calculate the singular value decomposition (SVD) of the matrix
(20 points)

$$D = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$$

(b) Calculate the pseudoinverse of the matrix D from part (a). (15 points)

7. *Graphs and Incidence Matrices* (25 points total)

(a) What is the incidence matrix for the graph: (10 points)



(b) Perform elimination on the incidence matrix from part (a). (10 points)

- (c) What is the spanning tree for the graph from part (a) determined by the reduction of the incidence matrix calculated in part (b)? [Draw It] (5 points)

8. *Markov Matrices* (20 points total)

(a) What are the eigenvalues of the Markov matrix (10 points)

$$M = \begin{pmatrix} .5 & .7 \\ .5 & .3 \end{pmatrix}$$

Hint - It's not necessary to find the roots of the characteristic equation! Just use what you know about eigenvalues of positive Markov matrices, and how the sum of the eigenvalues relates to the trace of the matrix.

(b) What is the steady-state solution for the Markov matrix M from part (a)? (10 points)