Math 2270 - Practice Exam 4

University of Utah

Fall 2012

Name: ____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. *Cofactor Matrices* (15 points)

   Calculate the cofactor matrix of A:

$$A = \begin{pmatrix} 4 & 3 \\ 2 & 8 \end{pmatrix}$$

2. *Eigenvalues* (20 points)

Find the eigenvalues and the corresponding eigenvectors of

$$A = \begin{pmatrix} 2 & -12 \\ 1 & -5 \end{pmatrix}$$

3. *Diagonalization* (20 points)

   Diagonalize the matrix

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

4. *Positive Definite Matrices* (10 points)

   Prove that if A is positive definite, and B is positive definite, then $A + B$ is positive definite. (Hint - A matrix is positive definite if $\mathbf{x}^T A\mathbf{x} > 0$ for all $\mathbf{x} \neq 0$.)

5. *Jordan Form*

If a matrix has eigenvalues $\lambda = 2, 2, 1, 0$ what are all the possible Jordan forms of the matrix?

6. *Singular Value Decomposition* (20 points)

Calculate the singular value decomposition of the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

7. *Linear Transformations* (15 points)

(a) Is the transformation $T : \mathbb{R}^2 \to \mathbb{R}^1$ defined by

$$T(v_1, v_2) = 2v_1 + v_2$$

a linear transformation? Explain why or provide a counter-example.

(b) Is the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$T(v_1, v_2) \to (2v_1, 3v_1 + 2v_2, v_1 v_2)$$

a linear transformation? Explain why or provide a counter-example.