Math 2270 - Exam 4

University of Utah

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This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. *Cofactor Matrices* (20 points)

   Calculate the cofactor matrix of A:

   $$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{pmatrix}$$

2. *Eigenvalues* (20 points)

Find the eigenvalues and the corresponding eigenvectors of

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

3. *Diagonalization* (20 points)

Diagonalize the matrix

$$A = \begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix}$$

4. *Positive Definite Matrices* (10 points)

Prove that if R is a matrix with independent columns then $R^T R$ is positive definite. (Hint - A matrix is positive definite if $\mathbf{x}^T A \mathbf{x} > 0$ for all $\mathbf{x} \neq \mathbf{0}$.)

5. *Jordan Form* (10 points)

   If a matrix has eigenvalues $\lambda = 2, 2, 2, 1$ what are all the possible Jordan forms of the matrix?

6. *Singular Value Decomposition* (20 points)

Calculate the singular value decomposition of the matrix

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$$