Math 2270 - Exam 3

University of Utah

Fall 2012

Name: ____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (9 Points) *The Four Subspaces*

   - (3 points) The nullspace is the orthogonal complement of the ________________________.
   - (3 points) If an $m \times n$ matrix has a d-dimensional column space, then its left nullspace has dimension ________________________.
   - (3 points) For an $m \times n$ matrix A calculate:

$$dim(\mathbf{C}(A)) + dim(\mathbf{N}(A)) + dim(\mathbf{C}(A^T)) + dim(\mathbf{N}(A^T)).$$

2. (8 points) *Orthogonal Complements* - Find a basis for the orthogonal complement of the vector space

$$span \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

3. (20 points) *Projections* - Find the projection of the vector **b** onto the column space of the matrix A:

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 2 \end{pmatrix}.$$

4. (15 points) *Linear Regression* - Find the least squares regression line through the points $(1, 0)$, $(2, 1)$, and $(3, 3)$.

5. (15 points) *Orthonormal Bases* - Find an orthonormal basis for the vector space

$$span\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\2\\0\end{pmatrix},\begin{pmatrix}0\\1\\2\end{pmatrix}\right\}.$$

6. (8 points) *Properties of Matrices* - Using the three fundamental properties of the determinant (determinant of the identity is 1, switching rows switches the sign, linearity for each row) prove that a matrix with a row of 0s has determinant 0.

7. (15 points) *Calculating Determinants* - Calculate the determinant of the matrix

$$\begin{pmatrix} 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 1 & -1 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix}$$

8. (10 points) *Other Ways of Calculating Determinants* - Calculate the determinant of the matrix

$$\begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 2 & 0 & 4 \end{pmatrix}$$

using a cofactor expansion along row 3.