Math 2270 - Practice Exam 2

University of Utah

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This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (15 Points) *Subspaces*

   For the following subsets of $\mathbb{R}^n$ explain why they are or are not sub-spaces of $\mathbb{R}^n$:

   (a) The set of all linear combinations of two vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$.

   (b) The set of all vectors with first component equal to 2.

   (c) The set of all vectors with first component equal to 0.

2. (20 points) For the following matrix, determine the special solutions
   for the nullspace, calculate the nullspace, and give its dimension.

$$A = \begin{pmatrix} 1 & 2 & 2 & 4 \\ 3 & 8 & 6 & 16 \end{pmatrix}.$$

3. (20 points) Fow the following matrix, calculate the rank, determine the dimension of the column space, and provide a basis for the column space:

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix}.$$

4. (15 points) Calculate the dimension, and find a basis for, the vector space given by:

$$span \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\}.$$

5. (30 points) Calculate the complete solution $A\mathbf{x} = \mathbf{b}$ for the following:

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 2 & 6 & 9 \\ -1 & -3 & 3 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}.$$