Math 2270 - Exam 2

University of Utah

Fall 2012

Name: ____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (15 Points) *Subspaces*

   Circle the subsets below that are subspaces of $\mathbb{R}^3$:

   (a) The set of all vectors $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ with $a_3 = a_1 + a_2$.

   (b) $span \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$.

   (c) The set of all vectors $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ with $a_3 \geq a_1 + a_2$.

   (d) The set of all vectors $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ with $a_3 = 0$.

   (e) The set of all vectors $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ with $a_3 = 1$.

2. (20 points) For the following matrix, determine the special solutions for the nullspace, provide a basis for the nullspace, and state the dimension of the nullspace.

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{pmatrix}.$$

3. (20 points) Calculate the rank of the following matrix, provide a basis for the column space, and state the dimension of the column space:

$$B = \begin{pmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{pmatrix}.$$

4. (10 points) For an 8×11 matrix A of rank 5 what are:

- $dim(\mathbf{C}(A)) =$
- $dim(\mathbf{C}(A^T)) =$
- $dim(\mathbf{N}(A)) =$
- $dim(\mathbf{N}(A^T)) =$

5. (25 points) Calculate the complete solution $A\mathbf{x} = \mathbf{b}$ for the following:

$$A = \begin{pmatrix} 1 & 0 & -2 & 1 & 5 \\ 3 & 1 & -5 & 0 & 8 \\ 1 & 2 & 0 & -5 & -9 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 5 \\ 8 \\ -9 \end{pmatrix}.$$

6. (10 points) For what value b_3 does the following equation have a solution? (Don't worry about providing the solution, just give the necessary value of b_3.)

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 3 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ b_3 \end{pmatrix}.$$

7. (10 points extra credit) If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is a basis for a vector space $\mathbf{V}$ prove that every vector $\mathbf{v} \in \mathbf{V}$ can be expressed *uniquely* as a linear combination of the given basis vectors.