

# Math 2270 - Practice Exam 1

University of Utah

Fall 2012

Name: \_\_\_\_\_

1. (25 points) *Vector Basics*

For the vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

answer the following:

(a) (3 points)  $\mathbf{a} + \mathbf{b} =$

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

(b) (3 points)  $2\mathbf{a} =$

(c) (5 points)  $\|\mathbf{c}\| =$

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

(d) (5 points)  $\mathbf{a} \cdot \mathbf{b} =$

(e) (5 points) Give the components of a unit vector in the same direction as  $\mathbf{b}$ .

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

(f) (4 points) Do the three vectors span a line, a plane, or all of  $\mathbb{R}^3$ ?

2. (15 points) *Matrix Basics*

For the matrices

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 0 \end{pmatrix}$$

answer the following

(a) (3 points)  $A + B =$

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 0 \end{pmatrix}$$

**(b)** (3 points)  $A + C =$

**(c)** (3 points)  $AC =$

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 0 \end{pmatrix}$$

**(d)** (3 points)  $AB =$

**(e)** (3 points)  $BA =$

3. (20 points) *Systems of Equations*

Use elementary row operations to convert the system of equations

$$\begin{array}{rclcrcl} x & - & 2y & + & 3z & = & 9 \\ -x & + & 3y & & & = & -4 \\ 2x & - & 5y & + & 5z & = & 17 \end{array}$$

into upper-triangular form, and then use back-substitution to solve for the variables  $x, y, z$ . Be sure to show all your work.

4. (20 points) *Matrix Form and Inverses*

Write the system of linear equations from the last problem

$$\begin{array}{rclcl} x & - & 2y & + & 3z & = & 9 \\ -x & + & 3y & & & = & -4 \\ 2x & - & 5y & + & 5z & = & 17 \end{array}$$

in matrix form, and find the inverse of the coefficient matrix.



5. (20 points) *LU Decomposition*

Find the LU decomposition of the coefficient matrix for the system of linear equations

$$\begin{array}{rclcl} x & - & 2y & + & 3z & = & 9 \\ -x & + & 3y & & & = & -4 \\ 2x & - & 5y & + & 5z & = & 17 \end{array}$$