Math 2270 - Exam 1

University of Utah

Fall 2012

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (15 points) *Vector Basics*

   For the vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \mathbf{c} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

   answer the following, or explain why the question does not make sense:

   (a) (3 points) $2\mathbf{a} + 3\mathbf{c} =$

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \mathbf{c} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(b) (3 points) $||\mathbf{a}|| =$

(c) (2 points) What are the components of a unit vector in the same direction as **a**?

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \mathbf{c} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(d) (4 points) $\mathbf{b} \cdot \mathbf{c} =$

(e) (3 points) $\mathbf{a} \cdot \mathbf{b} \cdot \mathbf{c} =$

2. (10 points) *Matrix Basics*

   For the matrices

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 & 5 \\ 4 & 4 & 2 \\ 1 & 0 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

   answer the following, or explain why the question does not make sense:

   (a) (3 points) $A + C =$

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 & 5 \\ 4 & 4 & 2 \\ 1 & 0 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(b) (4 points) $CB =$

(c) (3 points) $BC =$

3. (15 points) *Elimination Issues*

(a) (5 points) For what value of a in the system of equations below does elimination fail to produce a unique solution?

$$\begin{array}{rcrcl} 3x & + & 2y & = & 10 \\ 6x & + & ay & = & b \end{array}$$

(b) (5 points) Given the determined value of a, for what value of b are there an infinite number of solutions?

(c) (5 points) For the determined values of a and b what are two distinct solutions?

4. (20 points) *Systems of Equations*

   Use elementary row operations to convert the system of equations

$$\begin{array}{rcrcrcr} 2x & + & 3y & + & 3z & = & 3 \\ 6x & + & 6y & + & 12z & = & 13 \\ 12x & + & 9y & - & z & = & 2 \end{array}$$

   into upper-triangular form, and then use back-substitution to solve for the variables x, y, z. Be sure to show all your work.

5. (15 points) *Inverting a Matrix*

   Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{pmatrix}$$

6. (15 points) *LDU Decomposition*

   Find the LDU decomposition of the matrix

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{pmatrix}$$

7. (10 points) *Symmetric Products*

   For the matrix

$$R = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \end{pmatrix}$$

   (a) (4 points) What is the transpose R^T?

   (b) (4 points) What is the symmetric product $R^T R$

   (c) (2 points) Does $R^T R = R R^T$?