Math 2270 - Assignment 7

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Section 3.5 - 1, 2, 3, 20, 28
Section 3.6 - 1, 3, 5, 11, 24

3.5 - Independence, Basis, and Dimension

3.5.1 Show that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are independent but $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are dependent:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \mathbf{v}_4 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}.$$

3.5.2 (Recommended) Find the largest possible number of independent vectors among

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\mathbf{v}_4 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \qquad \mathbf{v}_5 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{v}_6 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

3.5.3 Prove that if $a = 0$ or $d = 0$ or $f = 0$ (3 cases), the columns of U are dependent:

$$U = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}.$$

3.5.20 Find a basis for the plane $x - 2y + 3z = 0$ in $\mathbb{R}^3$. Then find a basis for the intersection if that plane with the xy plane. Then find a basis for all vectors perpendicular to the plane.

3.5.28 Find a basis for the space of all 2 by 3 matrices whose columns add to zero. Find a basis for the subspace whose rows also add to zero.

3.6 - Dimension of the Four Subspaces

3.6.1 **(a)** If a 7 by 9 matrix has rank 5, what are the dimensions of the four subspaces? What is the sum of all four dimensions?

(b) If a 3 by 4 matrix has rank 3, whare are its column space and left nullspace?

3.6.3 Find a basis for each of the four subspaces associated with A:

$$A = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

3.6.5 If $\mathbf{V}$ is the subspace spanned by $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, find a matrix A that has $\mathbf{V}$ as its row space. Find a matrix B that has $\mathbf{V}$ as its nullspace.

3.6.11 (Important) A is an m by n matrix of rank r. Suppose there are right sides $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has *no solution*.

(a) What are all inequalities ($<$ or $\leq$) that must be true between m, n and r?

(b) How do you know that $A^T\mathbf{y} = \mathbf{0}$ has solutions other than $\mathbf{y} = \mathbf{0}$?

3.6.24 (Important) $A^T\mathbf{y} = \mathbf{d}$ is solvable when **d** is in which of the four subspaces? The solution **y** is unique when the ________________________ contains only the zero vector.