Math 2270 - Assignment 6

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Section 3.3 - 1, 3, 17, 19, 22
Section 3.4 - 1, 4, 5, 6, 18

3.3 - The Rank and Row Reduced Form

3.3.1 Which of these rules gives a correct definition of the *rank* of A?

(a) The number of nonzero rows in R.

(b) The number of columns minus the total number of rows.

(c) The number of columns minus the number of free columns.

(d) The number of 1's in the matrix R.

3.3.3 Find the reduced R for each of these (block) matrices:

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 2 & 4 & 6 \end{pmatrix} \quad B = \begin{pmatrix} A & A \end{pmatrix} \quad C = \begin{pmatrix} A & A \\ A & 0 \end{pmatrix}$$

3.3.17 **(a)** Suppose column j of B is a combination of previous columns of B. Show that column j of AB is the same combination of previous columns of AB. Then AB cannot have new pivot columns, so **rank(AB)** $\leq$ **rank(B)**.

(b) Find A_1 and A_2 so that $rank(A_1 B) = 1$ and rank $A_2 B = 0$ for $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

3.3.19 (Important) Suppose A and B are n by n matrices, and $AB = I$. Prove from $rank(AB) \leq rank(A)$ that the rank of A is n. So A is invertible and B must be its two-sided inverse (section 2.5). Therefore $BA = I$ (*which is not so obvious!*).

3.3.22 Express A and then B as the sum of two rank one matrices:

$$\textbf{rank} = 2 \quad A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 4 \\ 1 & 1 & 8 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}.$$

3.4 - The Complete Solution to $A\mathbf{x} = \mathbf{b}$.

3.4.1 (Recommended) Execute the six steps of Worked Example **3.4 A** to describe the column space and nullspace of A and the complete solution $A\mathbf{x} = \mathbf{b}$:

$$A = \begin{pmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$$

3.4.4 Find the complete solution (also called the *general solution*) to

$$\begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

3.4.5 Under what condition on b_1, b_2, b_3 is the system solvable? Include **b** as a fourth column in elimination. Find all solution when that condition holds:

$$\begin{array}{rcrcrcl} x & + & 2y & - & 2z & = & b_1 \\ 2x & + & 5y & - & 4z & = & b_2 \\ 4x & + & 9y & - & 8z & = & b_3 \end{array} .$$

3.4.6 What conditions on b_1, b_2, b_3, b_4 make each system solvable? Find $\mathbf{x}$ in each case:

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 2 & 5 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & 9 & 12 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}.$$

3.4.18 Find by elimination the rank of A and also the rank of A^T:

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 2 & 11 & 5 \\ -1 & 2 & 10 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & q \end{pmatrix} \text{ (rank depends on } q\text{).}$$