Math 2270 - Assignment 5

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Section 3.1 - 1, 2, 10, 20, 23
Section 3.2 - 1, 2, 4, 18, 36

3.1 - Spaces of Vectors

In the definition of a vector space, vector addition $\mathbf{x} + \mathbf{y}$ and scalar multiplication $c\mathbf{x}$ must obey the following eight rules:

(1) $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$

(2) $\mathbf{x} + (\mathbf{y} + \mathbf{z}) = (\mathbf{x} + \mathbf{y}) + \mathbf{z}$

(3) There is a unique "zero vector" such that $\mathbf{x} + \mathbf{0} = \mathbf{x}$ for all $\mathbf{x}$

(4) For each $\mathbf{x}$ there is a unique vector $-\mathbf{x}$ such that $\mathbf{x} + (-\mathbf{x}) = \mathbf{0}$

(5) 1 times $\mathbf{x}$ equals $\mathbf{x}$

(6) $(c_1 c_2)\mathbf{x} = c_1(c_2\mathbf{x})$

(7) $c(\mathbf{x} + \mathbf{y}) = c\mathbf{x} + c\mathbf{y}$

(8) $(c_1 + c_2)\mathbf{x} = c_1\mathbf{x} + c_2\mathbf{x}$.

3.1.1 Suppose $(x_1, x_2) + (y_1, y_2)$ is defined to be $(x_1 + y_2, x_2 + y_1)$. With the usual multiplication $c\mathbf{x} = (cx_1, cx_2)$, which of the eight conditions are not satisfied?

3.1.2 Suppose the multiplication $c\mathbf{x}$ is defined to produce $(cx_1, 0)$ instead of (cx_1, cx_2). With the usual addition in $\mathbb{R}^2$ are the eight conditions satisfied?

3.1.10 Which of the following subsets of $\mathbb{R}^3$ are actually subspaces?

- **(a)** The plane of vectors (b_1, b_2, b_3) with $b_1 = b_2$.
- **(b)** The plane of vectors with $b_1 = 1$.
- **(c)** The vectors with $b_1 b_2 b_3 = 0$.
- **(d)** All linear combinations of $\mathbf{v} = (1, 4, 0)$ and $\mathbf{w} = (2, 2, 2)$.
- **(e)** All vectors that satisfy $b_1 + b_2 + b_3 = 0$.
- **(f)** All vectors with $b_1 \leq b_2 \leq b_3$.

3.1.20 For which right sides (find a condition on b_1, b_2, b_3) are these systems solvable?

(a) $$\begin{pmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

(b) $$\begin{pmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

3.1.23 If we add an extra column $\mathbf{b}$ to a matrix A, then the column space gets larger unless ____________. Give an example where the column space gets larger and an example where it doesn't. Why is $A\mathbf{x} = \mathbf{b}$ solvable exactly when the column space *doesn't* ger larger - it is the same for A and $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$?

3.2 - The Nullspace of A: Solving $A\mathbf{x} = \mathbf{b}$

3.2.1 Reduce the matrices to their ordinary echelon forms U:

(a) $A = \begin{pmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{pmatrix}$

(b) $B = \begin{pmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{pmatrix}$.

3.2.2 For the matrices in Problem 3.2.1, find a special solution for each free variable. (Set the free variable equal to 1. Set the other free variables equal to zero.)

3.2.4 By further row operations on each U in Problem 3.2.1, find the reduced echelon form R. *True or false*: The nullspace of R equals the nullspace of U.

3.2.18 The plane $x - 3y - z = 12$ is parallel to the plane $x - 3y - z = 0$ in Problem 3.2.17. One particular point on this plane is $(12, 0, 0)$. All points on the plan have the form (fill in the first components)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} \\ 0 \\ 1 \end{pmatrix}.$$

3.2.36 How is the nullspace $\mathbf{N}(C)$ related to the spaces $\mathbf{N}(A)$ and $\mathbf{N}(B)$, if $C = \begin{pmatrix} A \\ B \end{pmatrix}$?