Math 2270 - Assignment 3

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Section 2.3 - 1,2,3,7,17
Section 2.4 - 1,2,13,14,32

1 Section 2.3 - Elimination Using Matrices

2.3.1 Write down the 3 by 3 matrices that produce these elimination steps:

(a) E_{21} subtracts 5 times row 1 from row 2.

(b) E_{32} subtracts -7 times row 2 from row 3.

(c) P exchanges rows 1 and 2, then rows 2 and 3.

a) $E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) $E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7 & 1 \end{pmatrix}$

c) $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$(12)(23) = (132)$

2.3.2 In Problem 1, applying E_{21} and then E_{32} to $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ gives

$$E_{32}E_{21}\mathbf{b} = \begin{pmatrix} 1 \\ -5 \\ 35 \end{pmatrix}.$$

Applying E_{32} before E_{21} gives

$$E_{21}E_{32}\mathbf{b} = \begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix}.$$

When E_{32} comes first, row 3 feels no effect from row 1.

2.3.3 Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix} \quad \text{and} \quad E_{32}E_{31}E_{21}A = U.$$

$$\boxed{E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \qquad \boxed{E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}}$$

$$\boxed{E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}}$$

$$E_{32}E_{31}E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{pmatrix} = U}$$

2.3.7 Suppose E subtracts 7 times row 1 from row 3.

(a) To *invert* that step you should ___add___ 7 times row ___1___ to row ___3___.

(b) What "inverse matrix" E^{-1} takes the reverse step (so $E^{-1}E = I$)?

(c) If the reverse step is applied first (and then E) show that $EE^{-1} = I$.

b) $$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{pmatrix} \quad \boxed{E^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{pmatrix}}$$

c) $$EE^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.3.17 The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4)$ and $(2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns (a, b, c).

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 8 \\ 14 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 8 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 8 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \Rightarrow c = 1,\ b = 1,\ a = 2$$

$$\boxed{y = x^2 + x + 2}$$

2 Section 2.4 - Rules for Matrix Operations

2.4.1 A is a 3 by 5, B is a 5 by 3, C is a 5 by 1. D is 3 by 1 *All entries are* 1. Which of these matrix operations are allowed, and what are the results

$$BA \qquad AB \qquad ABD \qquad DBA \qquad A(B+C).$$

$$AB\ \cancel{BA} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{pmatrix}$$

$$BA\ \cancel{AB} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \end{pmatrix}$$

~~ABD is not allowed.~~

$$ABD = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 15 \\ 15 \\ 15 \end{pmatrix}$$

~~DBA~~ ~~DBA~~

DBA is not allowed.

A(B+C) is not allowed. (We'd be adding a 3x3 to a 3x1.)

2.4.2 What rows or columns or matrices do you multiply to find

- **(a)** the third column of AB?
- **(b)** the first row of AB?
- **(c)** the entry in row 3, column 4 of AB?
- **(d)** the entry in row 1, column 1 of CDE?

a) The matrix A by the third column of B.

b) The first row of A by the matrix B.

c) The third row of A by the fourth column of B.

d) The product of the first row of C with the matrix D and the first column of E.

2.4.13 Which of the following matrices are guaranteed to equal $(A - B)^2$:

$$A^2 - B^2,$$
$$(B - A)^2,$$
$$A^2 - 2AB + B^2,$$
$$A(A - B) - B(A - B),$$
$$A^2 - AB - BA + B^2?$$

Circled entries are guaranteed to be equal.

2.4.14 True or false:

(a) If A^2 is defined then A is necessarily square.

(b) if AB and BA are defined then A and B are square.

(c) If AB and BA are defined then AB and BA are square.

(d) If $AB = B$ then $A = I$.

a) True.

b) False. If AB and BA are defined then if A is $m \times n$ then B is $n \times m$. Could be that $m = n$, but not necessarily.

c) True.

d) False. For example, if

$$A = B = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}.$$

2.4.32 (*Very important*) Suppose you solve $A\mathbf{x} = \mathbf{b}$ for three special right sides $\mathbf{b}$:

$$A\mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } A\mathbf{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } A\mathbf{x}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

If the three solutions $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ are the columns of a matrix X, what is A times X?

$$AX = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$