Math 2270 - Assignment 3

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Section 2.3 - 1,2,3,7,17
Section 2.4 - 1,2,13,14,32

1 Section 2.3 - Elimination Using Matrices

2.3.1 Write down the 3 by 3 matrices that produce these elimination steps:

(a) E_{21} subtracts 5 times row 1 from row 2.

(b) E_{32} subtracts -7 times row 2 from row 3.

(c) P exchanges rows 1 and 2, then rows 2 and 3.

2.3.2 In Problem 1, applying E_{21} and then E_{32} to $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ gives

$$E_{32}E_{21}\mathbf{b} = \underline{\hspace{3cm}}.$$

Applying E_{32} before E_{21} gives

$$E_{21}E_{32}\mathbf{b} = \underline{\hspace{3cm}}.$$

When E_{32} comes first, row ______________ feels no effect from row ______________.

2.3.3 Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix} \qquad \text{and} \qquad E_{32}E_{31}E_{21}A = U.$$

2.3.7 Suppose E subtracts 7 times row 1 from row 3.

(a) To *invert* that step you should ______________ 7 times row ______________ to row ______________.

(b) What "inverse matrix" E^{-1} takes the reverse step (so $E^{-1}E = I$)?

(c) If the reverse step is applied first (and then E) show that $EE^{-1} = I$.

2.3.17 The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4)$ and $(2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns (a, b, c).

2 Section 2.4 - Rules for Matrix Operations

2.4.1 A is a 3 by 5, B is a 5 by 3, C is a 5 by 1, and D is 3 by 1. *All entries are* 1. Which of these matrix operations are allowed, and what are the results

$$BA \qquad AB \qquad ABD \qquad DBA \qquad A(B+C).$$

2.4.2 What rows or columns or matrices do you multiply to find

(a) the third column of AB?

(b) the first row of AB?

(c) the entry in row 3, column 4 of AB?

(d) the entry in row 1, column 1 of CDE?

2.4.13 Which of the following matrices are guaranteed to equal $(A - B)^2$:

$$A^2 - B^2,$$
$$(B - A)^2,$$
$$A^2 - 2AB + B^2,$$
$$A(A - B) - B(A - B),$$
$$A^2 - AB - BA + B^2?$$

2.4.14 True or false:

- **(a)** If A^2 is defined then A is necessarily square.
- **(b)** if AB and BA are defined then A and B are square.
- **(c)** If AB and BA are defined then AB and BA are square.
- **(d)** If $AB = B$ then $A = I$.

2.4.32 (*Very important*) Suppose you solve $A\mathbf{x} = \mathbf{b}$ for three special right sides $\mathbf{b}$:

$$A\mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } A\mathbf{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } A\mathbf{x}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

If the three solutions $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ are the columns of a matrix X, what is A times X?