Math 2270 - Assignment 2

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Section 2.1 - 4,5,9,13,17
Section 2.2 - 3,6,11,12,19

1 Section 2.1 - Vectors and Linear Equations

2.1.4 Find a point with $z = 2$ on the intersection line of the planes $x + y + 3z = 6$ and $x - y + z = 4$. Find the point with $z = 0$. Find a third point halfway between.

2.1.5 The first of these equations plus the second equals the third:

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 2 \\ x & + & 2y & + & z & = & 3 \\ 2x & + & 3y & + & 2z & = & 5 \end{array}$$

The first two planes meet along a line. The third plane contains that line, because if x, y, z satisfy the first two equations then they also ________________. The equations have infinitely many solutions (the whole line **L**). Find three solutions on **L**.

2.1.9 Compute each $A\mathbf{x}$ by dot products of the rows with the column vector:

(a) $$\begin{pmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}.$$

(b) $$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}.$$

2.1.13 **(a)** A matrix with m rows and n columns multiplies a vector with ______________ components to produce a vector with ______________ components.

(b) The planes from the m equations $A\mathbf{x} = \mathbf{b}$ are in ______________-dimensional space. The combination of the columns of A is in ______________-dimensional space.

2.1.17 Find the matrix P that multiplies $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ to give $\begin{pmatrix} y \\ z \\ x \end{pmatrix}$. Find the matrix Q that multiplies $\begin{pmatrix} y \\ z \\ x \end{pmatrix}$ to bring back $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

2 Section 2.2 - The Idea of Elimination

2.2.3 What multiple of equations 1 should be *subtracted* from equation 2?

$$\begin{array}{rcrcl} 2x & - & 4y & = & 6 \\ -x & + & 5y & = & 0 \end{array}.$$

After this elimination step, solve the triangular system. If the right side changes to $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$, what is the new solution?

2.2.6 Choose a coefficient b that makes this system singular. Then choose a right side g that makes it solvable. Find two solutions in that singular case,

$$\begin{array}{rcrcl} 2x & + & by & = & 16 \\ 4x & + & 8y & = & g \end{array}.$$

2.2.11 (Recommended) A system of linear equations can't have exactly two solutions. Why?[1]

(a) If $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ are two solutions, what is another solution?

(b) If 25 planes meet at two points, where else do they meet?

[1] You're not begin asked to answer "why" here. Parts (a) and (b) lead you through an explanation as to why.

2.2.12 Reduce this system to upper triangular form by two row operations

$$\begin{array}{rcrcrcr} 2x & + & 3y & + & z & = & 8 \\ 4x & + & 7y & + & 5z & = & 20 \\ & - & 2y & + & 2z & = & 0 \end{array} .$$

Circle the pivots. Solve by back substitution for z, y, x.

2.2.19 Which number q makes this system singular and which right side t gives it infinitely many solutions? Find the solution that has $z = 1$.

$$\begin{array}{ccccccc} x & + & 4y & - & 2z & = & 1 \\ x & + & 7y & - & 6z & = & 6 \\ & & 3y & + & qz & = & t \end{array}.$$