Math 2270 - Assignment 14

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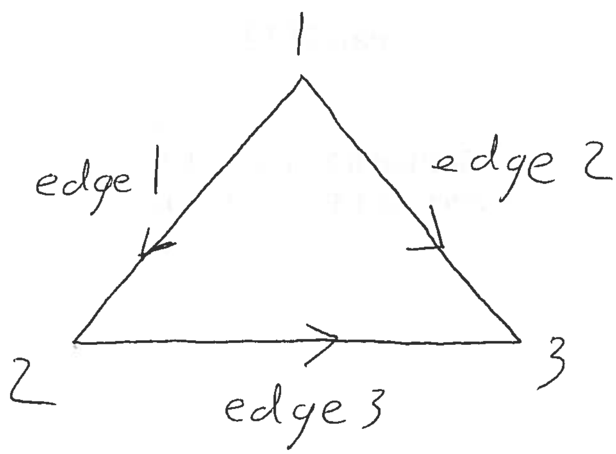
Section 8.2 - 1, 2, 3, 4, 5
Section 8.3 - 1, 2, 3, 9, 10

8.2 - Graphs and Networks

8.2.1 - Write down the 3×3 incidence matrix A for the triangle graph. The first row has -1 in column 1 and $+1$ in column 2. What vectors (x_1, x_2, x_3) are in its nullspace? How do you know that $(1, 0, 0)$ is not in its row space?

The triangle graph looks like:



8.2.2 - Write down A^T for the triangle graph. Find a vector **y** in its nullspace. The components of **y** are current on the edges - how much current is going around the triangle?

8.2.3 - Eliminate x_1 and x_2 from the third equation to find the echelon matrix U. What tree corresponds to the two nonzero rows of U?

$$\begin{array}{rcrcl} -x_1 & + & x_2 & = & b_1 \\ -x_1 & + & x_3 & = & b_2 \\ -x_2 & + & x_3 & = & b_3 \end{array}.$$

8.2.4 - Choose a vector (b_1, b_2, b_3) for which $A\mathbf{x} = \mathbf{b}$ can be solved, and another vector $\mathbf{b}$ that allows no solution. How are those $\mathbf{b}$'s related to $\mathbf{y} = (1, -1, 1)$?

8.2.5 - Choose a vector (f_1, f_2, f_3) for which $A^T\mathbf{y} = \mathbf{f}$ can be solved, and a vector $\mathbf{f}$ that allows no solution. How are those $\mathbf{f}$'s related to $\mathbf{x} = (1, 1, 1)$? The equation $A^T\mathbf{y} = \mathbf{f}$ is Kirchoff's ____________ law.

Markov Matrices, Population, and Economics

8.3.1 - Find the eigenvalues of this Markov matrix (their sum is the trace):

$$A = \begin{pmatrix} .90 & .15 \\ .10 & .85 \end{pmatrix}$$

8.3.2 - Diagonalize the Markov matrix in Problem 1 to $A = S\Lambda S^{-1}$ by finding its other eigenvector:

$$A = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} 1 & \\ & .75 \end{pmatrix} \begin{pmatrix} & \\ & \end{pmatrix}.$$

What is the limit of $A^k = S\Lambda^k S^{-1}$ when $\Lambda^k = \begin{pmatrix} 1 & 0 \\ 0 & .75^k \end{pmatrix}$ approaches $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$?

8.3.3 - What are the eigenvalues and steady state eigenvectors for these Markov matrices?

$$A = \begin{pmatrix} 1 & .2 \\ 0 & .8 \end{pmatrix} \qquad A = \begin{pmatrix} .2 & 1 \\ .8 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}.$$

8.3.9 - Prove that the square of a Markov matrix is also a Markov matrix.

8.3.10 If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a Markov matrix, its eigenvalues are 1 and ____________. The steady state eigenvector is $\mathbf{x}_1 =$ ____________.