Math 2270 - Assignment 10

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Section 5.2 - 1, 3, 11, 15, 16
Section 5.3 - 1, 6, 7, 8, 16

5.2 - Permutations and Cofactors

5.2.1 Compute the determinants of A, B, C from six terms.[1] Are their rows independent?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

[1] Using the big formula.

5.2.3 Show that $det(A) = 0$ regardless of the five nonzeros marked by x's:

$$A = \begin{pmatrix} x & x & x \\ 0 & 0 & x \\ 0 & 0 & x \end{pmatrix}$$

What are the cofactors of row 1?

What is the rank of A?

What are the 6 terms in the big formula for $det(A)$?

5.2.11 Find all the cofactors and put them into cofactor matrices C, D. Find AC and $det(B)$.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{pmatrix}.$$

5.2.15 The tridiagonal $1, 1, 1$ matrix of order n has determinant E_n:

$$E_1 = |1| \qquad E_2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$E_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \qquad E_4 = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}.$$

(a) By cofactors show that $E_n = E_{n-1} - E_{n-2}$.

(b) Starting from $E_1 = 1$ and $E_2 = 0$ find $E_3, E_4, \ldots, E_8$.

(c) By noticing how these numbers eventually repeat, find E_{100}.

5.2.16 F_n is the determinant of the $1, 1, -1$ tridiagonal matrix of order n:

$$F_2 = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} \quad F_3 = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 3$$

$$F_4 = \begin{vmatrix} 1 & -1 & & \\ 1 & 1 & -1 & \\ & 1 & 1 & -1 \\ & & 1 & 1 \end{vmatrix} \neq 4.$$

Expand in cofactors to show that $F_n = F_{n-1} + F_{n-2}$. These determinants are *Fibonacci numbers* $1, 2, 3, 5, 8, 13, \ldots$. The sequence ususally starts $1, 1, 2, 3$ (with two $1'$s) so our F_n is the usual F_{n+1}.

5.3 - Cramer's Rule, Inverses, and Volumes

5.3.1 Solve these linear equations by Cramer's rule $x_j = \frac{det(B_j)}{det(A)}$:

(a) $$\begin{array}{rcrcl} 2x_1 & + & 5x_2 & = & 1 \\ x_1 & + & 4x_2 & = & 2 \end{array}$$

(b) $$\begin{array}{rcrcrcl} 2x_1 & + & x_2 & & & = & 1 \\ x_1 & + & 2x_2 & + & x_3 & = & 0 \\ & & x_2 & + & 2x_3 & = & 0 \end{array}.$$

5.3.6 Find A^{-1} from the cofactor formula $C^T/det(A)$. Use symmetry in part (b).

(a) $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 7 & 1 \end{pmatrix}$

(b) $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$

5.3.7 If all the cofactors are zero, how do you know that A has no inverse? If none of the cofactors are zero, is A sure to be invertible?

5.3.8 Find the cofactors of A and multiply AC^T to find $det(A)$:

$$A = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 6 & -3 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

and $AC^T =$

If you change that 4 to 100, why is $det(A)$ unchanged?

5.3.16 **(a)** Find the area of the parallelogram with edges $\mathbf{v} = (3, 2)$ and $\mathbf{w} = (1, 4)$.

(b) Find the area of the triangle with sides $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{v} + \mathbf{w}$. Draw it.

(c) Find the area of the triangle with sides $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{w} - \mathbf{v}$. Draw it.