Math 2270 - Assignment 1

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Section 1.1 - 1,2,13,16,30
Section 1.2 - 1,2,3,27,29
Section 1.3 - 1,2,6,8,13

1 Section 1.1 - Vectors and Linear Combinations

1.1.1 Describe geometrically (line, plane, or all of $\mathbb{R}^3$) all linear combinations of

(a) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, and $\begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}$.

(b) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$.

(c) $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$, and $\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$.

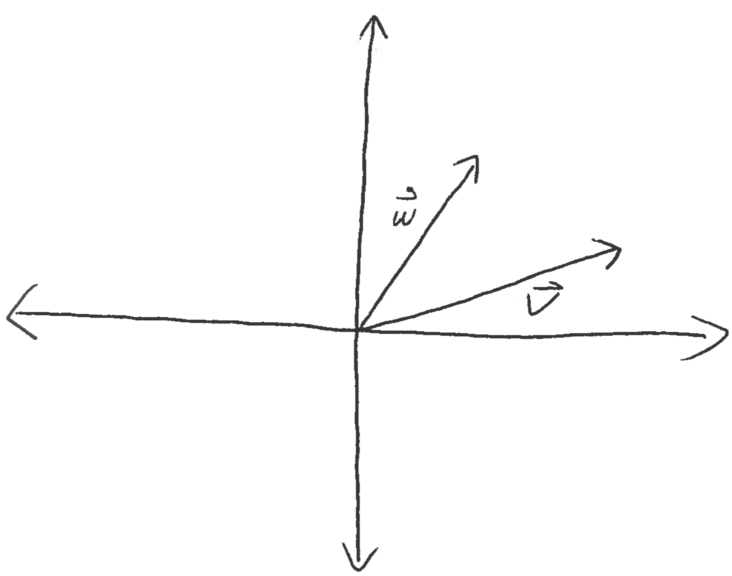
1.1.2 Draw $\mathbf{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ and $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$ in a single xy plane.

1.1.13 **(a)** What is the sum $\mathbf{V}$ of the twelve vectors that go from the center of a clock to the hours 1:00, 2:00, ..., 12:00?

(b) If the 2:00 vector is removed, why do the 11 remaining vectors add to 8:00?

(c) What are the components of that 2:00 vector $\mathbf{v} = (\cos\theta, \sin\theta)$?

1.1.16 Mark the point $-\mathbf{v} + 2\mathbf{w}$ and any other combination $c\mathbf{v} + d\mathbf{w}$ with $c + d = 1$. Draw the line of all combinations that have $c + d = 1$.



1.1.30 The linear combinations of $\mathbf{v} = (a, b)$ and $\mathbf{w} = (c, d)$ fill the plane unless ______________. Find four vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{z}$ with four components each so that their combinations $c\mathbf{u} + d\mathbf{v} + e\mathbf{w} + f\mathbf{z}$ produce all vectors (b_1, b_2, b_3, b_4) in four-dimensional space.

2 Section 1.2 - Lengths and Dot Products

1.2.1 Calculate the dot products $\mathbf{u} \cdot \mathbf{v}$ and $\mathbf{u} \cdot \mathbf{w}$ and $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ and $\mathbf{w} \cdot \mathbf{v}$:

$$\mathbf{u} = \begin{pmatrix} -.6 \\ .8 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}.$$

1.2.2 Compute the lengths $||\mathbf{u}||$ and $||\mathbf{v}||$ and $||\mathbf{w}||$ of those vectors. Check the Schwarz inequalities $|\mathbf{u} \cdot \mathbf{v}| \leq ||\mathbf{u}|| ||\mathbf{v}||$ and $|\mathbf{v} \cdot \mathbf{w}| \leq ||\mathbf{v}|| ||\mathbf{w}||$.

1.2.3 Find unit vectors in the directions of $\mathbf{v}$ and $\mathbf{w}$ in Problem 1, and the cosine of the angle θ. Choose vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ that make $0°$, $90°$, and $180°$ angles with $\mathbf{w}$.

1.2.27 (*Recommended*) If $||\mathbf{v}|| = 5$ and $||\mathbf{w}|| = 3$, what are the smallest and largest values of $||\mathbf{v} - \mathbf{w}||$? What are the smallest and largest values of $\mathbf{v} \cdot \mathbf{w}$?

1.2.29 Pick any numbers that add to $x + y + z = 0$. Find the angle between your vector $\mathbf{v} = (x, y, z)$ and the vector $\mathbf{w} = (z, x, y)$. Challenge question: Explain why $\mathbf{v} \cdot \mathbf{w}/||\mathbf{v}||||\mathbf{w}||$ is always $-\frac{1}{2}$.

3 Section 1.3 - Matrices

1.3.1 Find the linear combinations $2\mathbf{s}_1 + 3\mathbf{s}_2 + 4\mathbf{s}_3 = \mathbf{b}$. Then write $\mathbf{b}$ as a matrix-vector multiplication $S\mathbf{x}$. Compute the dot products (row of S)$\cdot\mathbf{x}$:

$$\mathbf{s}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{s}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \mathbf{s}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ go into the columns of } S.$$

1.3.2 Solve these equations $S\mathbf{y} = \mathbf{b}$ with $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3$ in the columns of S:

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}.$$

The sum of the first n odd numbers is ______________.

1.3.6 Which values of c give dependent columns (combination equals zero)?

$$\begin{pmatrix} 1 & 3 & 5 \\ 1 & 2 & 4 \\ 1 & 1 & c \end{pmatrix}, \begin{pmatrix} 1 & 0 & c \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \begin{pmatrix} c & c & c \\ 2 & 1 & 5 \\ 3 & 3 & 6 \end{pmatrix}.$$

1.3.8 Moving to a 4 by 4 difference equation $A\mathbf{x} = \mathbf{b}$, find the four components x_1, x_2, x_3, x_4. Then write this solution as $\mathbf{x} = S\mathbf{b}$ to find the inverse matrix $S = A^{-1}$:

$$A\mathbf{x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \mathbf{b}.$$

1.3.13 The very last words of worked example 1.3B say that the 5 by 5 centered difference matrix *is not* invertible. Write down the 5 equations $C\mathbf{x} = \mathbf{b}$. Find the combination of left sides that gives zero. What combination of b_1, b_2, b_3, b_4, b_5 must be zero? (The 5 columns lie on a "4-dimensional hyperplane" in 5-dimensional space.)