

Math 1010 - Lecture 9 Notes

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In today's lecture we'll extend our notions of solving algebraic equalities and inequalities to equations involving absolute values.

1 Absolute Value Equations

Suppose we're given the absolute value equation:

$$|x| = 12.$$

What this equation is asking, in words, is "for what numbers x is the absolute value of the number equal to 12?" In this case there will be two solutions, namely -12 and 12 , both of which have absolute value 12.

This was pretty easy, but it illustrates this concept that, when dealing with an absolute value equation, we could possibly (and in fact frequently) have more than one solution.

Let's work through a slightly more difficult question. Suppose we're given the equation:

$$|x + 3| + 2 = 9$$

and we want to find the values of x for which this equation is true. In other words, the values of x that satisfy this equation. Well, to figure this out we'd first subtract 2 from both sides to get:

$$|x + 3| = 7.$$

Now, this will be true if the absolute value of $x + 3$ is equal to 7. In other words, this will be true if $x + 3$ is equal to either 7 *or* -7 . So, to find our solutions we'd set up:

$$|x + 3| = 7$$

which means either

$$x + 3 = 7 \quad \text{or} \quad x + 3 = -7.$$

So, to find our possible solutions, we'd solve each of these equations individually, to get $x = 4$ and $x = -10$, respectively. These x -values, 4 and -10 , will be the two solutions to our equation.

It's frequently, but not always, the case that we get two solutions. We can have situations with more than two solutions¹, only one solution, or even zero solutions. For example, the equation:

$$|x| = 0$$

has only one solution, namely $x = 0$.

On the other hand, the equation:

$$|x| = -2$$

has **no** solution, as the absolute value of a number cannot be negative.

¹We won't see any of these in this section, as they'd involve equations that are more complicated than the ones we'll be examining.

Examples

Solve the following absolute value equations:

1. $|5x - 3| + 8 = 22.$

2. $|4 - 3x| = 0.$

3. $|5 - 2x| + 10 = 6.$

2 Equations with Absolute Values on Both Sides

So far we've just dealt with equations with one absolute value. We can also deal with equations with more than one absolute value. For example, suppose we're given the equation:

$$|10 - 3x| = |x + 7|.$$

This will be true if either $10 - 3x = x + 7$ or $10 - 3x = -(x + 7)$. So, the two equations could be equal, or they could be *opposites*. In either case, they would have the same absolute value.

So, to solve this we'd need to break it up into its two cases.

Case I - Both sides are equal

$$10 - 3x = x + 7;$$

add $3x$ to both sides,

$$10 = 4x + 7;$$

subtract 7 from both sides,

$$3 = 4x;$$

divide both sides by 4,

$$\frac{3}{4} = x.$$

Case II - The sides are opposite

$$10 - 3x = -(x + 7)$$

$$\rightarrow 10 - 3x = -x - 7;$$

add $3x$ to both sides,

$$10 = 2x - 7;$$

add 7 to both sides,

$$17 = 2x;$$

divide both sides by 2,

$$\frac{17}{2} = x.$$

So, our two solutions (the two values of x that satisfy the equation) are $x = 3/4$ and $x = 17/2$.

Examples

Solve for x in the following equations.

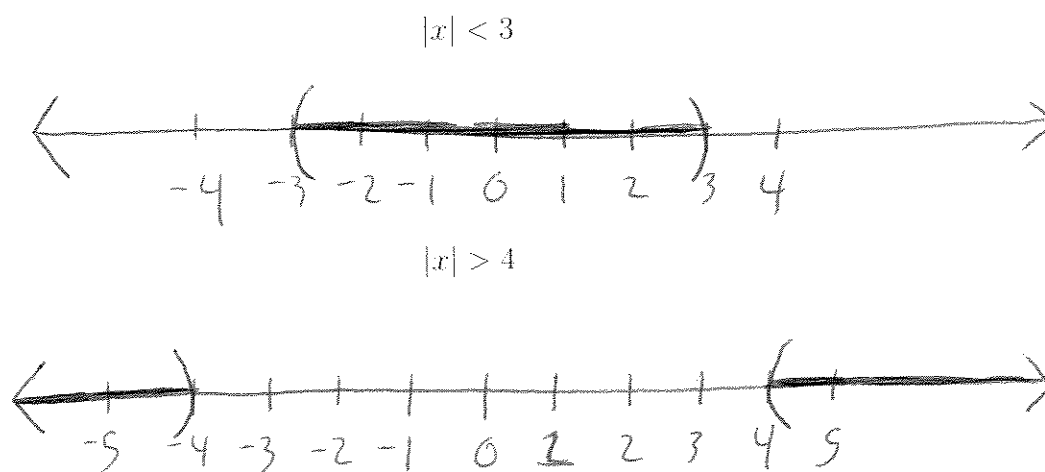
1. $3|2x - 5| + 4 = 7.$

2. $|2x + 7| = |2x + 9|$.

3 Absolute Value Inequalities

Thus far we've only dealt with equalities involving absolute values. We can also have inequalities with absolute values.

For example, the possible values of x that satisfy the inequality below are plotted on the number line:



In general, the solutions of $|x| < a$ are all values of x that lie between $-a$ and a . In other words, the values of x such that $-a < x < a$.

On the other hand, the solutions of $|x| > b$ are all values of x that are greater than b or less than $-b$. In other words, the values of x such that either $x > b$ or $x < -b$.

We note that the above relations hold if we replace $<$ with \leq , and/or $>$ with \geq .

Examples

Solve the following inequalities, and graph the possible solutions on the number line.

1. $|7 - 2h| \geq 9$.

2. $|3x + 10| < -1$.

3. $\left| \frac{2x - 4}{5} \right| - 9 \leq 3.$